A MINIMUM COST DESIGN
FOR AN AUTOMATED WAREHOUSE

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A MINIMUM COST DESIGN

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NOMENCLATURE

AS/RS - Automated Storage Retrieval System

BHEHT - Building height

BLTH - Building length

BWTH - Building width

Class Based TOR Storage - Dedicated Storage

DC - dual command (an S/R trip performed on a dual command basis)

DEPTH - unit load depth

Full TOR Storage - An extreme case of dedicated storage where every pallet is assigned to a particular opening

HEHT - unit load height

hv - horizontal travel velocity of the S/R

ICOL - denotes the rack length in number of columns

NOAI - number of storage aisles

NOL - denotes the rack height in number of levels.

SC - single command (an S/R trip performed on a single command basis)

S/R - Storage/Retrieval Machine

TNOA - Total number of openings in the warehouse

vv - vertical travel velocity of the S/R

WT - the amount of time an order spends waiting in the queue (waiting time)

WTH - unit load width

λ - the throughput level demanded from the system.

SUMMARY

This study is concerned with the development of an algorithm for constructing a minimum cost automated warehouse. The cost elements considered include the land cost, the building cost, the rack cost, the storage/retrieval machine cost and the annual maintenance cost associated with the building and S/R's. The primary decision variables are the number of aisles, the number of levels and the number of columns in each aisle.

Two important variables that affect the final solution are the total number of openings required and the throughput level demanded from the system. The former has been analyzed for a particular inventory system. A method is presented for determining the total storage requirements of the warehouse based on the available inventory records. The approach is demonstrated within the framework of a hypothetical example.

The throughput capacity of the system is a function of both the S/R travel time and the storage method. Hence, the mean and variance of the S/R travel time for single and dual command have been analyzed for both randomized and dedicated storage methods. Closed form expressions to compute S/R travel time are presented for randomized storage. In addition, single and dual command travel time distributions are derived for randomized storage.

Combining the above findings with the algorithm to minimize total present worth cost offers the user a method to arrive at a final design

that satisfies the throughput requirement while minimizing cost. Lastly, sensitivity analysis has been performed for those variables the user has to provide.

CHAPTER I

INTRODUCTION

Automated high-rise warehousing systems are having a dramatic effect on the design and operation of large capacity, high volume storage facilities. High-rise storage is one of the fastest growing areas of material handling. It has the ability to store thousands of unit loads and retrieve any particular one within a few minutes, when under computer control. Such systems make it possible to totally integrate material handling and storage into the manufacturing and distribution processes. The installations that are reported to be successfully operating have provided considerable savings in terms of space, time, manpower and the physical and inventory control over all materials in process or in storage.

Objective

The primary objective of this study is to develop an algorithm to aid in the design of automated warehouses. The purpose is to give the user the ability to quickly develop final designs and their associated costs for different parameters. It is necessary to understand that the algorithm does not provide the detailed design. With the high number of design options provided by numerous firms, it would have been impossible to consider the detailed design and still maintain the flexibility of the approach. Hence, the user should view the resulting cost as a benchmark to compare various design alternatives. In developing the algorithm, care has been exercised to keep it as general as possible.

Importance of the Study

This study examines automated high-rise warehousing systems. Economic justification for such systems is often complex. Labor savings alone are usually not enough. Some other factors that are often included in the justification are:

- Lower building cost
- Better space utilization
- Improved inventory control
- Lower land cost
- Increased throughput
- Less damage to stored material
- Increased automation opportunities

Automated storage and retrieval systems generally require considerable investment in capital and tend to be more inflexible than manually operated conventional storage systems. Estimating the system performance and the associated costs becomes extremely important in making investment decisions for such systems. Hence, a quantitative approach is essential for decision making.

In the light of the above fact, an attempt was made to learn more about the decision process employed by various firms operating in this field. At the 1978 Material Handling Show (sponsored by the Material Handling Institute, Inc.) 34 firms joined the exhibition as automated handling system manufacturers. Several among these firms were actually

AS/RS manufacturers. To the author's knowledge, none of these firms had a general purpose design package. Instead, most of them reported that they had various design procedures based on judgement and past experience. Furthermore, these procedures were tailored to the type of equipment manufactured by the firm.

Foundation Material

Various designs are available for high-rise storage systems. This study considers one basic type. Product(s) is stored in pallet racks whose height is a decision variable. Each storage/retrieval machine (S/R) operates in a single aisle with pallet racks on either side. Figure 1-1 portrays an S/R traveling between two rows of racks. The configuration of one S/R and two rows of racks is defined as one aisle of storage. The aisle consists of a number of columns (up to L); height is indicated by the number of levels (up to H). Thus, the total number of pallet loads stored in one aisle is equal to 2 x H x L (assuming one load per opening). Only one S/R accesses a given rack. It moves vertically along a support and the support moves horizontally along a track. Both motions can occur at the same time so that the S/R moves diagonally. Thus, travel time to a particular opening is determined by the maximum of the horizontal and vertical travel times. There are two options available for the S/R itself. It can either be a fully automated S/R with no operator or one with a man-on-board. In the first case the S/R receives instructions via the computer; in the latter case, the operator picks up a list of orders from the input-output point and operates the S/R accordingly. The operator or the S/R

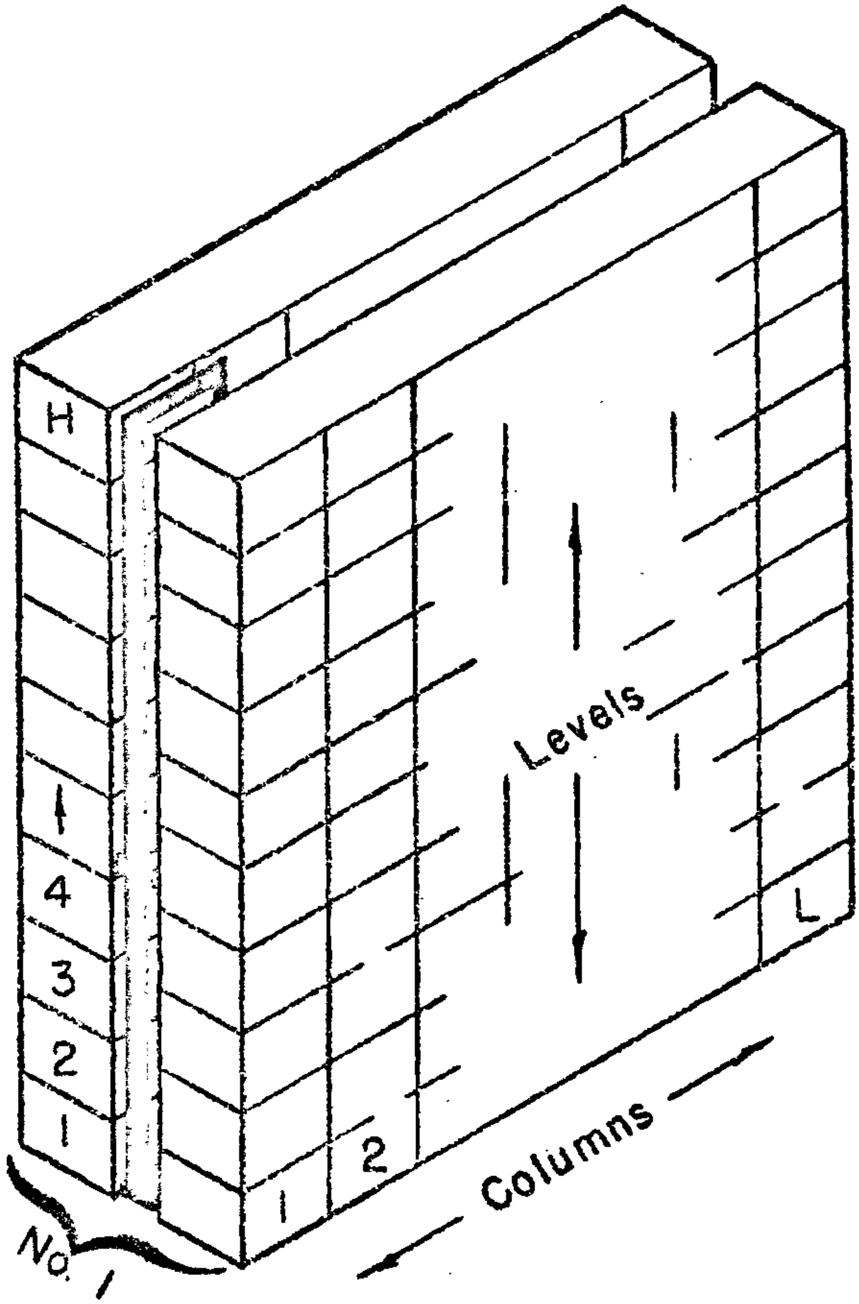

Figure 1-1. One Aisle of Storage.
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permission of the author.

may access the product in either of two ways. The shuttle or forks can move in and out to either side to pick up full or empty pallets. Since the aisle is only a little wider than the S/R, the S/R cannot turn around while traveling back to the origin. The origin, also called the input-output (I/O) point, the pick up and delivery (P/D) point, etc. consists of space for a few pallets to queue outside the aisle at a level low enough for either fork trucks or conveyor I/O. The fork trucks or conveyors deliver product for storage in the racks and pick up product for shipping.

A number of terms remain to be defined. The first two are single command and dual command. Single command is used simply to refer to the storage or retrieval of one pallet load. The S/R starts from the I/O point, travels to the required opening, stores or retrieves the pallet load and returns to the origin. Dual command involves two pallet loads in the following manner. The S/R leaves the I/O point with a load to be stored, travels to the particular opening, stores the load, then travels to another opening from which a load is retrieved, and returns to the origin. As can be seen, the dual command calls for simultaneous storage and retrieval orders.

Two additional terms encountered in the warehousing area are dedicated and randomized storage. Some use randomized storage to refer to the situation where the location for a load to be stored is the closest open location and a first in, first out retrieval policy is used. Such a policy is called "randomized storage" since, at any point in time, the distribution of pallet loads of various products over the rack looks as if they were randomly allocated to the openings. In this thesis, randomized storage refers to the situation in which both storage positions and retrieval positions are selected on a purely random basis.

Dedicated storage is used when each opening is assigned to a particular product and storage/retrieval is performed randomly within a given set of openings assigned to one product. As will be seen later, assumptions relating to single-dual command travel time computations and type of storage method used have important effects on system throughput.

This study has made the following assumptions:

1. Each pallet contains only one item type, i.e., pallet assignment (the assignment of multiple items to the same pallet) will not be analyzed.

2. It is assumed that all openings are the same size. One pallet load is stored in each opening.

3. The storage orders are performed on the FCFS basis. The retrievals are on a FIFO basis.

4. Dual cycles will be performed on a percent basis (the percent of S/R trips made on a dual basis is to be provided by the user).

5. The S/R operates either on a single or on a dual command basis, i.e. multiple stops are not allowed.

6. Each item is carried in every aisle. Therefore, transfer cars are not considered, i.e. there is one S/R per aisle.

7. Each aisle is equally likely to be selected when a storage and/or retrieval order arrives.

8. Decision variables such as unit load dimensions, unit load weight, and S/R logic and travel velocities are assumed to be provided by the user. The decision variables considered are the number of aisles, number of levels, and the number of columns.

Description of the Problem

Assuming that the unit load dimensions, the total number of slots required, S/R travel velocities and values for other parameters to be discussed later are given, the problem is to find the design that will minimize the total present worth cost, yet satisfy the throughput requirement. The final design will indicate the number of aisles and the number of levels and columns in each aisle. It will also provide the system throughput capacity with and without including the waiting time associated with each storage/retrieval operation. In addition to these variables, the building dimensions for the resulting design are provided.

One side-problem encountered has been the computation of the expected travel time of the S/R in filling storage/retrieval orders. As stated before, travel time is crucial in determining the throughput capacity of a given design. Hence, a separate chapter (Chapter IV) treats the analysis of travel time.

Literature Search

Earliest Research Efforts

Previous publications on AS/R systems date back to the late 1960's with the subject of interest being the stacker cranes. Except for those which describe successful applications, there are very few articles (10, 11, 14, 17, 18) that review the advances made in this area.

As indicated in Bafna's (2) literature survey, prior to 1972 there have been several attempts to develop a design methodology for stacker crane systems. One of the earliest known attempts to be made was by Thompson and Cnossen (15) at the Ford Motor Company. They developed a simulation model for a computer controlled warehouse. Their study is reported to be a model which shows how the warehouse would perform under alternative operation rules. The major drawback of their study is that it did not consider the cost of the system. Another simulation study appeared in late 1969. The program was developed by the Storage Systems Department of Clark Equipment Company; Schwind (14) has only made mention of it. Bafna (2) reports this simulator to be insufficient for two reasons. First, it does not simulate a complete system but only the aisles associated with one S/R. Secondly, the simulator is not tied to the program which yields the cost of the system. Bafna (2) also presents an analytical model developed by one of the firms operating in this area. It consists of a formula to find the optimum number of aisles, N, given the fraction of total inventory moved in and out per day, the number of openings visited by the crane per trip, the depth of stacking, the horizontal travel speed of the crane, the required fork cycles per trip and the fork cycle time. However, how it takes into account the various cost elements involved in such an analysis is not mentioned. In addition, the system throughput is not considered and above all, the height of storage (the principle advantage of these systems) is ignored. Bafna (2) concludes the survey by stating that there has not been much work done in this area.

Following Bafna (2), research in the automated warehousing area has increased. It can be grouped under two main categories: that done by firms which manufacture AS/R systems and by the Material Handling Institute (MHI); and that done in the universities.

Recent Research: Commercial Sector

Work done by private firms addresses two areas. The first is the analysis of S/R travel time under various circumstances and the second covers economic justification for these systems.

Sand (13) has analyzed the S/R travel time for a rack with fixed picking area and randomized reserve storage. However, she has not considered turnover-based storage assignment. She assumes a fixed picking area with two pallet openings per product and random reserve area. First she presents a two part heuristic that minimizes the number of picking lots and then minimizes the travel time for each lot. Then, assuming that the picking area is square in time, the average travel time to the reserve area is found. However, she does not complete the analysis to find total travel time for replenishment, stating that it is not straightforward. She concluded her study by indicating that further research is necessary for analyzing storage (putaway) and possibilities for dual cycles (interleaving).

Barrett (3) used a simulation model to evaluate the effectiveness of various rules for order picking systems that allow multiple stops for each crane trip into the aisle. Four heuristic methods are investigated for their ability to reduce the number of picking lots. The model tests system performance for three factors: storage assignment (the assignment of pallet loads to storage locations), lot assignment (the assignment of an order to a picking lot) and batch quantity (the number of orders from which picking lots are formed). System performance is measured by the number of lots, percent of minimum lots, number of stops and travel time (comparisons are given for inventories with different 'ABC' characteristics and customer ordering patterns). For combined effects of lot assignment and batch quantity it compares the effectiveness of the above mentioned four heuristics in regard to number of lots, percent of minimum lots and number of stops. From the standpoint of travel time it is stated that it can be reduced by 20% - 35% when turnover-based storage is used instead of random. However, all of the results are highly empirical.

The second area of research for private firms has been the investigation of the factors which effect the total cost of an AS/RS. One such study is due to Allred (1). It mainly covers the justification of automated storage systems, and the techniques for purchasing and constructing such systems. A simple hypothetical model is presented to compute the cost difference between a conventional and an automated storage system. (How the cost of the automated system has been computed is not stated). The two systems are compared on three bases: the tangible annual costs, less tangible annual costs, and the least tangible annual costs. Both systems are evaluated for the receiving, storage and order picking functions on the basis of material queue, paper work queue and average elapsed time. The study is concluded with a discussion on methods of system procurement.

Another study that develops a basis for estimating AS/RS investment cost is presented by Zollinger (19). He has compiled detailed cost information for more than 60 AS/RS installations. Assuming that the data related to the particular design under question is given, cost estimates are developed for the following elements:

1. Rack prices including installation and stacker support rail.
2. Storage machine price including controls, electrification, guiderail and installation.
3. Building price for conventional construction including services and sprinklers (construction height between 25' and 85').

In addition, a method of computing building dimensions is presented, including the required clearances to be added. The cost model used in this thesis is based on that of Zollinger (19). One slight modification has been the addition of land cost and recurring costs to the above list of cost elements. Hence, to avoid duplication, details of this approach are left to Chapter II.

The Material Handling Institute, Inc. (MHI) has several publications related to the study. However, they are mostly descriptive in nature. One of them (9) deserves mention; it has been developed by the member-companies of the AS/RS Product Section of the Material Handling Institute. It is a useful source for quick reference and discusses the following areas:

- system objectives
- system performance
- system engineering
- system control
- system hardware
- system justification
- system installation
- system acceptance procedure
- system warranty
- system maintenance
- system glossary

The considerations given in (9) are basically recommendations. They are intended only to provide guidelines for technical procedure and contain information that may be helpful to the purchasers and/or users of AS/R systems.

Rygh (12) discusses the common uses, configurations and benefits of four basic types of systems mentioned as unit load S/R systems, order-picking systems, in-process systems, and combined systems. Also, the effect of new computer technology on such systems is briefly explained.

Recent Research: University Sector

From the standpoint of work done at universities, there are very few published articles. Hausman, Schwarz and Graves have analyzed S/R travel times and published their findings in (5), (6) and (7).

In (5) it is assumed that the rack is square in time, i.e. the vertical and horizontal speeds are such that the time to reach the row most distant from the I/O point equals the time to reach the most distant column. In addition, interleaving (dual command) is ignored. Expected one-way travel is computed for the discrete case as follows: let,

R = number of rows in each rack,

C = number of columns in each rack,

N = number of pallet storage locations in one rack ($N = RC$),

y_i = the ranked one-way time for the S/R machine to travel from the I/O point to location i, $i = 1, \ldots, N$; ranked so that $y_i \leq y_{i+1}$, all $i < N$.

λ_j = the turnover of the item on pallet j. It is the number of times that the item on pallet j will be stored (and subsequently retrieved) per unit time. For convenience the λ_j are ranked so that $\lambda_j \geq \lambda_{j+1}$ for $j < N$.

Under random storage assignment the expected one-way travel time for a pallet, T_R, is:

$$T_R = \overline{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$$

With the highest turnover pallet assigned to the closest location, the expected one-way travel time is:

$$T_T = \frac{\sum_{i=1}^{N} y_i \lambda_i}{\sum_{i=1}^{N} \lambda_i}$$

To switch to the continuous representation of the rack, the travel time is first normalized. So, the travel time to a location in the k^{th} percentile is:

$$y(k) = k^{\frac{1}{2}} \quad 0 < k \leq 1$$

The turnover distribution is derived by making use of the "ABC" phenomenon for inventories and the basic EOQ model. The "ABC" curve is represented by the function:

$$G(i) = i^s \quad \text{for} \quad 0 < s \leq 1$$

where demand is measured in full pallet loads. Let:

D(i) = demand rate (pallets per unit time) of item i.

Q(i) = the economic order quantity of item i.

By definition

$$G(i) = i^s = \int_0^i D(j)\ dj \ / \int_0^1 D(j)dj$$

$$\text{Letting } \int_0^1 D(j)dj = 1,\ i^s = \int_0^i D(j)dj$$

which has the solution:

$$D(i) = si^{s-1} \quad 0 < i \leq 1$$

Since all items are ordered using the EOQ model, then $Q(i) = (2KD(i))^{\frac{1}{2}}$ where K is the ratio of order cost to holding cost. Hence the average turnover is: $2D(i)/Q(i) = (2D(i)/K)^{\frac{1}{2}}$.

In order to determine the time index, i, of the j^{th} pallet, the following equation has to be solved:

$$j = \int_0^{i(j)} ((2KD(k))^{\frac{1}{2}}/2)dk$$

If j is normalized by letting j = j/L (L = total number of rack locations required for the average inventory of all items), then

$$i(j) = j^{2/(s+1)}$$

Hence, $D_j^{'} = D_{i(j)} = sj^{2(s-1)/(s+1)}$. Therefore, the turnover of the j^{th} pallet is

$$\lambda(j) = (2D_j^{'}/K)^{\frac{1}{2}} = (2s/K)^{\frac{1}{2}}(j^{(s-1)/(s+1)})$$

With the above results, it is shown that the one-way travel time for randomized storage is:

$$T_R' = E\left[y(i)\right] = \int_0^1 i^{\frac{1}{2}}\, di = 2/3$$

and for full-turnover based storage it is:

$$T_T' = \frac{\int_{j=0}^{1} \lambda(j)y(j)dj}{\int_{j=0}^{1} \lambda(j)dj} = \frac{4s}{(5s + 1)}$$

It is also shown that for typical inventory distributions, the percentage reduction in S/R travel time ranges from 26% to 71% when full-turnover based storage is used instead of random.

Another storage assignment rule which is similar to full TOR based storage is class-based turnover (dedicated) storage. For a two class system with the Class I region to be used for the higher turnover pallets, and the Class II region for the lower turnover pallets, the one-way travel time is (see Figure 1-2):

$$T_2'(R) = \frac{\int_{j=0}^{R^2} \lambda(j)\overline{y}_I dj + \int_{j=R^2}^{1} \lambda(j)\overline{y}_{II} dj}{\int_0^1 \lambda(j)dj}$$

where R = the partitioning value

$\overline{y}_K$ = average travel time to region K.

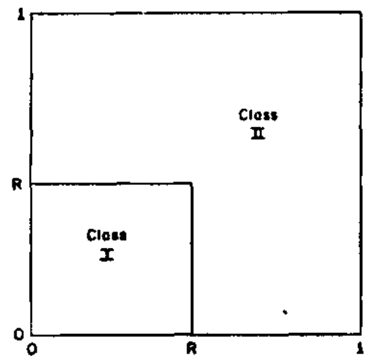


Figure 1-2. Two Class Storage Assignment
(Reprinted from (6))

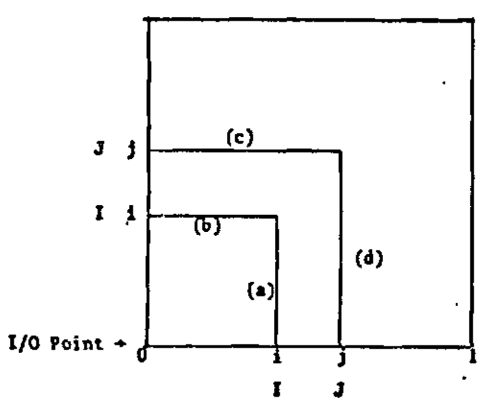


Figure 1-3. Representation of Continuous Rack Locations
(Reprinted from (7))

Since $\bar{y}_I = \frac{2}{3}R$ and

$$\bar{y}_{II} = \frac{2}{3}\ (1-R^3)\ /\ (1-R^2)$$

the above equation will reduce to

$$T_2'(R) = \frac{2}{3}\left[R^{(5s+1)/(s+1)} + (1-R^3)(1-R^{4s/(s+1)})\ /\ (1-R^2)\right]$$

The percentage inprovement over random storage for this case ranges between 18% and 53%. Of course, the percentage improvement increases as the skewness of the inventory distribution increases, i.e. as s approaches zero. Similar results are presented for a three class system.

This study is concluded with a comparison between the discrete and continuous representations of the rack. Four cases are included in the comparison: Random, Full Turnover, Two-Class, and Three-Class. The continuous approximation performs well only for the random storage case. For a 20%/60% inventory distribution (i.e. 20% of the items represent 60% of total demand) the continuous model underestimates the S/R travel time by 6%, 3.4%, and 7.2% for the full TOR, Two-Class and Three-Class systems respectively. Likewise, for a 20%/80% inventory distribution, the travel time is underestimated by 24.8%, 16%, and 22%. As the skewness increases, the underestimation gets considerably larger.

Following the above paper, Graves, Hausman and Schwarz published two complementary papers (6) and (7). The assumptions stated for (5) also hold for their subsequent studies, with the exception that interleaving (dual cycle) is not ignored. In (6) an expression is developed to find the expected interleave time between any two percentile sets of rack locations. Specifically, the expected S/R travel time from the set of rack locations i units from the I/O point to the set of rack locations j units from the I/O point is (see Figure 1-3):

$$E\left[d(i,j)\right] = \frac{1}{12ij}\left[i^3 - 9i^2j + 12ij^2\right] \quad \text{if } 2i < j$$

$$= \frac{1}{12ij}\left[17i^3 - 33i^2j + 24ij^2 - 2j^3\right] \quad \text{if } 2i > j \qquad (1\text{-}1)$$

where $0 \leq i < j \leq 1$.

The expected interleave time for the randomized storage method is determined using the following expression:

$$L = 2\int_{i=0}^{1}\int_{j=i}^{1} E\left[d(i,j)\right]\, f(i,j)\, dj\,di \qquad (1\text{-}2)$$

where f(i,j) is the joint density of travel from the set of locations i time units from the I/O point to the set of locations j time units from the I/O point. Using the above expression for $E\left[d(i,j)\right]$ and by decomposing the integral boundaries accordingly, L is found to be 7/15.

The expected interleave time is also developed for the full TOR based storage. Given full TOR, the probability that an item is within i time units of the I/O point, F(i), satisfies:

$$F(i) = \frac{\int_0^{i^2} \lambda\ (j)\ dj}{\int_0^1 \lambda\ (j)\ dj} = i^{\frac{4s}{s+1}}$$

Thus, $$f(i) = \frac{dF(i)}{di} = \frac{4s}{s+1}\, i^{\frac{4s}{s+1} - 1} \qquad 0 < i \le 1$$

or $$f(i) = 2zi^{2z-1} \quad \text{where } z = 2s/(s+1)$$

Consequently $f(i,j) = 4z^2(ij)^{2z - 1}$. When $f(i,j)$ is substituted in (1-2), with (1-1) used for $E\left[d(i,j)\right]$,

$$L = \frac{2z^2}{3}\left\{\frac{48z^3 + 36z^2 + 42z - 48 + 96(2^{-2z-2})}{(4z + 1)(2z + 2)(2z + 1)(2z)(2z - 1)}\right\} \qquad (1\text{-}3)$$

is obtained.

The expected interleave time in a "square-L" region is also determined. Assuming that storage/retrieval within the region is randomized, the expected interleave time $E\left[d(I,J)\right]$, is

$$E\left[d(I,J)\right] = 2 \int_{i=I}^{J} \int_{j=i}^{J} E\left[d(I,J)\right]\ f(i,j)\ djdi \qquad (1\text{-}4)$$

The probability of a store/retrieve within i units of the rack, F(i), is

$$F(i) = \frac{i^2 - I^2}{(J^2 - I^2)} \quad \text{for } I \le i \le J$$

Differentiating F(i) yields: $f(i) = \frac{2i}{(J^2 - I^2)}$ for $I \leq i \leq J$. Therefore, by the independence assumption:

$$f(i,j) = f(i)f(j) = \frac{4ij}{(J^2 - I^2)^2} \quad I \leq i,\ j \leq J \tag{1-5}$$

Now substitute (1-5) for f(i,j) and (1-1) for $E\left[d(I,J)\right]$ in (1-4). Evaluating the resulting expression in pieces gives:

$$\begin{aligned} E\left[d(I,J)\right] &= \tfrac{1}{30}(13J^3 + 36J^2I - 21JI^2 + 32I^3) \ / \ (J+I)^2 \quad \text{if } J < 2I \\ &= \tfrac{1}{30}(14J^5 - 40J^3I^2 + 30J^2I^3 - 5JI^4) \ / \ (J^2-I^2)^2 \text{if } J > 2I \end{aligned} \tag{1-6}$$

The study is also concerned with the determination of the expected interleave time, L, given two-class storage assignment. Conceptually, L is:

$$L = p(I,I)L_I + p(II,II)L_{II} + 2p(I,II)L_{I,II} \tag{1-7}$$

Under randomized storage (1-7) can be written as:

$$7/15 = R_1^4 \cdot L_I + (1 - R_1^2)^2 \cdot L_{II} + R_1^2(1 - R_1^2)L_{I,II} \tag{1-8}$$

L_I is found from (1-6) by setting I = 0 and J = R_1. Likewise, L_{II} is found by setting I = R_1 and J = 1 in (1-6). Hence, $L_{I,II}$ will be found from (1-8) as:

$$L_{I,II} = \frac{1}{60}\,(40 - 30R_1 + 5R_1^2 - 14R_1^3) \;/\; (1 - R_1^2) \quad \text{if } R_1 < \tfrac{1}{2}$$

$$= \frac{1}{60}\,(1 - 9R_1 + 71R_1^2 - 39R_1^3 + 46R_1^4) \;/\; (R_1^2(1 + R_1)) \quad \text{if } R_1 > \tfrac{1}{2} \tag{1-9}$$

Under full TOR, the probability of an item being within R_1 units of the I/O point, $F(R_1)$, is:

$$F(R_1) = R_1^{4s/(s+1)} = R_1^{2z}$$

Consequently,

$$p(I,I) = R_1^{4z}$$
$$p(II,II) = (1 - R_1^{2z})^2$$
$$\text{and} \quad p(I,II) = R_1^{2z}(1 - R_1^{2z})$$

Substituting the above probabilities into (1-7) with the corresponding expressions obtained for L_I, L_{II} and $L_{I,II}$, one obtains the following expression for L:

$$L = \left[(R_1^{2z})^2 (7R_1/15)\right] + \left[(1 - R_1^{2z})^2 \cdot \frac{1}{30}(14 - 40R_1^2 + 30R_1^3 - 5R_1^4) \;/\; (1 - R_1^2)^2\right]$$
$$+ 2\left[R_1^{2z}(1 - R_1^{2z}) \cdot \frac{1}{60}(40 - 30R_1 + 5R_1^2 - 14R_1^3) \;/\; (1 - R_1^2)\right] \quad \text{if } R_1 < \tfrac{1}{2}$$

$$L = \left[(R_1^{2z})^2 (7R_1/15)\right] + \left[(1 - R_1^{2z})^2 \cdot \frac{1}{30}(13 + 36R_1 - 21R_1^2 + 32R_1^3) \;/\; (1 + R_1)^2\right]$$
$$+ 2\left[R_1^{2z}(1 - R_1^{2z}) \cdot \frac{1}{60}(1 - 9R_1 + 71R_1^2 - 39R_1^3 + 46R_1^4) \;/\; (R_1^2(1 + R_1))\right]$$
$$\text{if } R_1 > \tfrac{1}{2}$$

Graves, et al, considered the case in which the retrieve portion of the dual cycle is not FCFS, instead the retrieve order is selected from a queue of length K. Let T_x be the expected round-trip time given that the store is in class X. Then the round-trip time, RT, is:

$$RT = P(I)T_I + P(II)T_{II} \tag{1-10}$$

It was previously shown that $p(I) = R_1^{2z}$ and $p(II) = 1 - R_1^{2z}$. Now consider T_I: the expected time for the store is the expected one-way time in class I, W_I. The expected time for the retrieve depends on whether or not an inspection of the first K retrieves yields a retrieve of class I. Let X represent the number of class I retrieves in the retrieve queue, $0 \leq X \leq K$. Then, given independence between stores and retrieves, as well as the default policy (if no match is found, select the first retrieve), X is a Markov-Chain whose steady-state probabilities, π_x, are derived and, after evaluating T_I and T_{II}, substituted in (1-10). The resulting expression for RT is quite complicated. The approach becomes much more tedious for three-class systems.

In (6) the analysis concludes with a treatment of the additional rack openings required in order to keep fixed the default probability (the probability of not being able to store an item in a desired location). It has been shown that the steady-state probability of n in the system is Poisson with parameter $m = \lambda/\mu$ (assuming a $M/G/\infty$ queue with expected arrival rate λ and expected service rate μ). Considering a rack with m + k locations, the problem is to find how large must k be in order to ensure that the probability of n exceeding (m + k) is some acceptably small number. For large numbers of openings the Poisson distribution with parameter m can be approximated by the Normal distribution. Consequently, the ratio of k to m, representing additional rack locations required as a fraction of m, is:

$$(k/m) = 2.575\ (c/m)^{\frac{1}{2}}$$

where c is the number of classes (classes are assumed to be equal size).

The companion paper, (7), extends the work done in (6). Using both the continuous and discrete approach, it compares the operating performance of several storage assignment/interleaving policies. The expected round-trip time is the expected time for the system to complete one store and one retrive. In mandatory interleaving (MIL) policies the expected round-trip time is twice the expected one-way time plus the expected interleave time. In no interleaving (NIL) policies, the expected round-trip time is four times the expected one-way travel time. Results are presented for the policies shown in Table 1-1.

Table 1-1. Storage Assignment/Interleaving Policies

1 - RAN/NIL/FCFS:	Random Storage Assignment; No Interleaving FCFS retrieve queue
2 - FULL/NIL/FCFS:	Full TOR based Storage Assignment; etc.
3 - C2/NIL/FCFS:	2-Class Storage Assignment; etc.
4 - C3/NIL/FCFS:	3-Class Storage Assignment; etc.
5 - RAN/MIL/FCFS:	Random Storage Assignment; Mandatory Interleaving; FCFS Retrieve Queue
6 - FULL/MIL/FCFS	
7 - C2/MIL/FCFS	
8 - C3/MIL/FCFS	
9 - C2/MIL/Q = K:	Selection queue of K retrieves
10 - C3/MIL/Q = K	

For a 20/80 inventory distribution, expected round trip time for above listed cases has been found as shown in Table 1-2 (discrete enumeration results are given in parenthesis; the third number denotes percent differences):

Table 1-2. Expected Round Trip Times

	Continuous Rack	Discrete Rack	Percent Difference
1	2.667	(2.664)	-.11
2	1.310	(1.640)	20.12
3	1.709	(1.933)	11.59
4	1.501	(1.774)	15.39
5	1.800	(1.795)	-.28
6	1.041	(1.223)	14.88
7	1.261	(1.390)	9.28
8	1.145	(1.296)	11.65
9	1.119	(1.328)	15.74
10	1.072	(1.247)	14.03

From the above results it can be readily seen that, except for randomized storage, the continuous model always underestimates the true answer. The percent underestimation varies between 9.28% and 20.12%.

Bafna (2) used a simulation model to estimate the values of the principle parameters of an AS/RS. The number of aisles, the rack height, the number of S/R's, and the horizontal and vertical velocities of the S/R's are the design variables. From the standpoint of transfer cars, it is assumed that any S/R having more than one aisle assigned to it must have a transfer car (a conservative assumption). The entire approach is composed of three stages: constructing the cost model, developing the simulation model and combining the cost model with the simulator in order to arrive at a design that minimizes total cost.

The cost model includes those costs that change with variations in the warehouse and S/R parameters. They include cost of floor space (land), cost of building, cost of racks, cost of S/R's, cost of transfer cars, and cost of fire protection. The equivalent annual cost of the system is found by computing and summing the capital recovery for each cost element. The annual labor cost is added to the above total in order to find the equivalent annual cost for the entire system. However, investment tax credits are ignored and the calculations are based on before tax returns.

The simulation model operates as follows: the initial values (lower limits) of the number of aisles, rack height, the number of S/R's and the horizontal and vertical travel speeds of the S/R's are provided by the user. The throughput is simulated with this initial set of variables. The calculated value of throughput is transferred to the main program and compared with the last throughput. If it has increased, a new series of variable sets are generated and the one giving the lowest annual cost is transferred to GASP to simulate the next throughput. If the throughput has not increased, the next set of variables from the series generated earlier is transferred to GASP and the throughput is calculated (the method by which a new series of variable value sets is generated is discussed in detail in (2)). This cycle is repeated until the calculated throughput satisfies the throughput demanded from the system. Every satisfactory throughput is resimulated with a longer simulation time to assure steady-state. If the revised value of throughput still exceeds the required level, simulation stops and final values of the variables are printed. Otherwise, the procedure described above starts repeating itself until the throughput requirement is satisfied.

The above described simulation model is analyzed with a sample run for an AS/RS with 5900 openings and 90 operations per hour throughput capacity. The solution is reached after 24 iterations. Required computer time is stated to be 243 decimal seconds on a CDC 6500 computer. The final horizontal and vertical velocities of the S/R's are equal to their initial values. Lastly, a sensitivity analysis is presented. The sensitivity of the cost to changes in the throughput of the system is analyzed. It has been found that the incremental cost per unit throughput is lower for higher throughputs. Other results reached by Bafna are subject to discussion and will be presented in Chapter VII.

Based on the literature survey, it can be concluded that previous analytical approaches failed to combine their results with an appropriate cost model and ignore any throughput requirement. Hence, this study has been performed to develop a general purpose design package based on analytical findings.

Summary

The purpose of this study is to develop an analytical approach to design automated warehouses. The objective is to minimize the total cost of the system, subject to the throughput requirement imposed over the system. The literature search does not indicate the existence of a similar study that has been published.

The above mentioned goal is reached in several stages. First, a method is presented for determining the total storage requirements based on inventory records (Chapter III). Chapter IV is concerned with the analysis of S/R travel time in regard to randomized and dedicated storage methods. The cost functions are analyzed in Chapter V. Based on this analysis and previous findings from Chapter IV, an algorithm that will minimize the total cost and satisfy the throughput requirement, is developed. Chapter VI is concerned with the discussion of sample run results and sensitivity analysis. Lastly, Chapter VII presents the conclusions and recommendations.

CHAPTER II

DEFINITION OF THE TOTAL PRESENT WORTH COST

Introduction

The total cost of an AS/RS is composed of numerous elements. They can be identified as the land cost, the building cost, the rack cost, and the S/R cost. These cost elements can be viewed as the initial cost of the system. Of course, there are other costs that can be classified as the initial cost. They include the required hardware, acquisition of the software, investment required in the service area, etc. However, such costs remain constant irrespective of the system design. Therefore, analyzing only those costs which vary with the design will serve the purpose of comparing alternative designs. Recurring costs, on the other hand, are composed of those costs associated with maintenence, direct personnel, secondary personnel, utilities, inventory carrying, etc. Once the total number of unit loads to be accommodated by the system is determined, such costs tend to show little variation with changes in the design, except for the maintenance cost on the building and on the S/R's. One other exception is the labor cost incurred if the S/R's are of the man-on-board type.*

*The costs obtained in this chapter are based on 1978 costs, verified by Zollinger in October 1978 and communicated by him in a private conversation at the Automated Material Handling Systems short course at Georgia Tech.

Cost of Land

The importance of land cost will vary with the land price. Depending on the location of the warehouse, it may have a considerable effect on the final design. The total area taken up by the warehouse will simply be the base area of the building. Let

BLTH = building length (in feet)

BWTH = building width (in feet)

LCOST = unit land price ($/square foot)

Then total land cost, say, C_L = BLTH x BWTH x LCOST dollars. A method of computing BLTH and BWTH will be presented under Building Cost.

Cost of Building

The method used to compute the building cost is referred to as the square-foot-floor-area method. Hence, as for the land cost, the base area of the building is required to compute the building cost.

The length and width of the building is found by using tables provided by Zollinger (19). Let

WTH = unit load width (in inches)

HEHT = unit load height (in inches)

DEPTH = unit load depth (in inches)

ICOL = number of columns of storage

NOAI = number of aisles

Then

$$\text{Rack length} = \text{HDST} = (\text{WTH} + 8'') \times 1/12 \times \text{ICOL (in feet)}$$

$$\text{Rack height} = \text{VDST} = (\text{HEHT} + 10'') \times 1/12 \times \text{NOL (in feet)}$$

$$\text{Storage width} = S_w = 3 \times (\text{DEPTH} + 6'') \times 1/12 \times \text{NOAI (in feet)}$$

Storage width is found assuming that sprinklers are provided. Next, the building dimensions can be found by adding the proper clearances.

$$\text{Building length} = \text{BLTH} = \text{HDST} + \text{IADD}'$$

$$\text{Building height} = \text{BHEHT} = \text{VDST} + 4.0'$$

$$\text{Building width (not rack supported)} = \text{BWTH} = S_w + 2.0'$$

where

IADD = an estimate of the space required for P/D stations, S/R extension beyond the end of the rack and conveyor or truck aisle. The amount of allowance depends upon the unit load width. Zollinger (19) provides an estimate as shown in Table 2-1. These allowances are given assuming no transfer cars. For practical and programming purposes, the following regression line was derived from the table.

$$\text{IADD} = 12.504243 + 0.447478 \text{ WTH}$$

The 4.0' added to building height is due to the fact that the lower storage level cannot be at floor level and the ceiling is not put directly on the top of the upper storage level. Thus, an allowance of 18" from the floor level and an allowance of 20" above the upper storage level has to be added. (Sprinklers are assumed to be in the flue spacing.)

The 2.0' added to building width is the allowance provided for the space between the walls and rack. This allowance need not be added if the building is rack supported.

Table 2-1. Minimum Building Length Addition.*

ADD TO RACK LENGTH

Pallet Length	System Without Transfer	System With Transfer
30"	26'	42'
36"	29'	45'
40"	30'	47'
42"	31'	48'
48"	34'	52'
52"	36'	54'

Table 2-2. Conversion Factors for Unit Building Cost.*

CLEAR HEIGHT (ft.)	FACTOR NUMBER
25	1.00
40	1.25
55	1.50
70	1.90
85	2.50

Table 2-3. Factor Numbers to Determine Rack Cost.*

LOAD VARIABLE	FACTOR NUMBER		
	1	2	3
Load Size (cu.ft.)	40	80	120
Load Weight (lbs.)	500	3000	6000
Rack Height (loads)	4	10	24

*Taken from (19) with permission of the author.

One remaining variable required to compute building cost is BCOST. That is, the cost per square foot of the building base assuming conventional construction (not rack supported) and including sprinklers. Zollinger (19) has proposed the following approach. Let BCOST be the cost incurred per square foot in building a 25' high building. The base cost will increase with the height of the building. Thus, if 25' is taken as the base, Table 2-2 shows the factor numbers to be multiplied by BCOST in order to determine the corresponding cost per square foot. The factor will subsequently be referred to as CFR (the conversion factor). Again for practical and programming purposes the following regression line will be used to find CFR:

$$\text{CFR} = 0.986508 - 0.005349\ \text{BHEHT} + 0.0002698\ (\text{BHEHT})^2$$

Hence, the building cost is obtained using the following expression:

$$\text{Building Cost} = C_B = \text{BLTH} \times \text{BWTH} \times \text{BCOST} \times \text{CFR}$$

Rack supported warehouses have increased in use. They bring various advantages. As given in (16)

> A rack supported warehouse is supported entirely or at least partially by the actual rack structure. The weight of the warehouse roof and metal sidings or skins to enclose the rack structure normally does not significantly increase the load support characteristics of the racks. As an example, if unit loads weighing from 2,000 to 4,000 pounds are stored in a rack structure to a height of from 8 to 10 tiers, the floor loading due to the rack

and its contents can exceed 1,000 pounds per square foot. If the rack must also support the skins and roof, another 70 pounds per square foot must be accommodated by the storage rack. Thus, the rack supported building requires storage racks with the capability of handling a 7% increase in the load handled by conventional storage racks.

Cost savings for rack supported buildings include the reduction of redundant steel structures for supporting the roof and siding. Erection cost is usually decreased since the roof and siding are added after the rack structure is complete. One estimate of the savings in fabricating and erecting a rack supported warehouse is that it can be as much as $6 per square foot less than a conventional warehouse with free-standing rack.

In addition to the significant savings in the price of the rack supported warehouse, there exists another cost savings which can be even more significant - tax savings! A rack supported warehouse qualifies as equipment due to its special purpose design. Hence, a writeoff period of 8 - 10 years, rather than 50 - 60 years, can be used. Furthermore, since the rack supported warehouse is treated as equipment, it qualifies for the investment tax credit.

Hence, if the building is going to be rack supported, BCOST x CFR will be decreased by 6.0. Thus,

$$\text{Building Cost} = C_B = \text{BLTH x BWTH X } \left[(\text{BCOST x CFR}) - 6.0\right]$$

Rack supported buildings also provide tax savings, as discussed subsequently.

Cost of Racks

The function of the racks is to support multiple levels of unit loads. For rack supported buildings they also support the skin and roof of the building.

There are various options for designing racks, depending upon the type of unit load and storage module. Conventional AS/RS racks have load support rails with space between them for shuttle clearance. Conventional pallet racks have load beams parallel to the aisle. Taking all forces into account and performing the stress analysis will dictate the type of design required. Including external forces for rack supported buildings is especially important.

Zollinger (19) presents the following approach for estimating the rack cost. It includes the cost of installation and S/R support rail. Table 2-3 shows the corresponding factor values based on the load size, the load weight, and the rack height expressed in number of unit loads. The factor values are added together and then multiplied by $14 to find the cost per rack opening. The figure obtained is then multiplied by the total number of openings in the system to obtain the total rack cost. The following example is provided: the unit load is 40" x 48" x 48" (depth x width x height) = 53.2 cubic feet. Load weight is 2,000 lbs, rack height is 10 levels. Thus, the factor adds up to 1 + 2 + 2 = 5. Hence, the cost per rack opening is 5 x $14 = $70.

An alternative approach to factor value addition is to use a regression line. White (16) provides the following expression:

$$\text{Cost/rack opening} = \$14\left[0.92484 + 0.025x + 0.0004424y - \frac{y^2}{82{,}500{,}000} + 0.23328z - 0.00476z^2\right]$$

where x = number of cubic feet in a unit load

y = weight of the unit load in pounds

z = height of the rack expressed in unit loads

For the above example this line gives $69.27 per opening. Hence, if we let

TNOA = total number of openings

ALS = unit load size (cu.ft.)

WEGT = weight of unit load (lbs)

then

$$TNOA = NOL \times ICOL \times 2.0 \times NOAI$$

$$ALS = (DEPTH \times WTH \times HEHT)/1728.0$$

$$RCOST = 0.92484 + (0.025 \times ALS) + (0.0004424 \times WEGT) - \frac{(WEGT)^2}{82{,}500{,}000}$$

Therefore, $RCOS = \left[RCOST + (0.23328 \times NOL) - (0.00476 \times NOL^2)\right]$ $\times\ 14.0$ and, Total Rack Cost = C_R = RCOS x TNOA.

Cost of S/R Machines

The basic function of the S/R machine, as the name implies, is to perform the storage/retrieval orders. Apart from other types of material handling equipment it is capable of performing high lifts, fast movements, and accurate positioning. It consists of three mechanical drives. The first is the horizontal drive which moves the machine back and forth along the aisle. The second is the hoist drive which raises and lowers the carriage along the height of the rack. The third drive is the shuttle drive which moves the shuttle in and out of the rack openings while transferring the load. It also powers the shuttle for transferring the load at the P/D station.

Based on Zollinger (19), the S/R cost is a function of the height of the AS/RS, the weight of the unit load, and the type and location of the control logic. If the height of the AS/RS is less than 30' then a base cost of \$13,000 per S/R applies; if the AS/RS is in the range from 30' to 42' then an additional cost of \$13,000 is added to each S/R; for heights from 42' to 60' an additional cost of \$26,000 is added to the base cost for each S/R; and for heights from 60' to 85' an incremental cost of \$39,000 is incurred (above the base cost).

If the load weighs less than 1000 lbs, then \$13,000 is contributed to the cost per S/R; for weight in the interval from 1000 lbs. to 3500 lbs., \$26,000 is contributed to the cost per S/R; for loads weighing from 3500 lbs. to 6500 lbs., \$39,000 is contributed to the cost per S/R; and for loads above 6500 lbs., \$52,000 is added to the overall cost of each S/R.

If the control logic is on-board a cost of \$26,000 is contributed to the cost of each S/R; if the control logic is off the machine, a cost of \$39,000 is contributed per S/R; and if a central console is used to control all S/R's, a cost of \$52,000 is contributed to each S/R.

As will be noticed the range of the AS/RS height and load weight for each interval is quite wide. Thus, the use of a regression line for this case will not provide a good approximation.

The Annual Labor Cost

The annual labor cost is included in the analysis if man-on-board S/R's are used. It is assumed that there is one operator on each S/R. Despite the increased labor cost, the cost per S/R will decrease due to reduction of controls on the machine. Recall that if control logic is on-board a cost of \$26,000 is contributed to the cost of each S/R. If man-on-board S/R is used, then instead of \$26,000, \$13,000 is added to the cost of each S/R[1]. Also, if we let

LBCOST = annual labor cost on a per operator basis

then, total annual labor cost, C_{LB}, will be:

$$C_{LB} = \text{LBCOST} \times \text{NOAI} \quad (\$/\text{yr})$$

The Annual Maintenance Cost

The annual maintenance cost is composed of the maintenance cost on the building and the S/R's. The building maintenance cost will be found as follows, let:

BMCOST = the annual maintenance cost per sq. ft. of the building base (to be provided by the user). Then, total building maintenance cost, C_{MB}, will be:

$$C_{MB} = \text{BLTH} \times \text{BWTH} \times \text{BMCOST} \quad \text{dollars/yr.}$$

Furthermore, if we let:

SRMCOST = the annual maintenance cost per S/R (to be provided by the user), then, the total S/R maintenance cost, C_{MSR}, will be:

[1]Above approach has been developed through verbal communication with Mr. H.A. Zollinger.

$$C_{MSR} = \text{NOAI} \times \text{SRMCOST dollars/yr.}$$

(assuming one S/R per aisle.)

Hence, the total annual maintenance cost, C_M, will be:

$$C_M = C_{MB} + C_{MSR} \quad \text{dollars/yr.}$$

Present Worth Cost of the System

The present worth cost of the system will be computed in two different ways, depending upon whether the building is rack supported or not. The following assumptions have been made in determining the present worth cost of the non-rack supported building:

1 - The planning horizon is ten years

2 - Land has no depreciation

3 - The S/R's, the rack, and the controls have a ten year write-off period, and no salvage value

4 - The building has a forty year write-off period and the salvage value equals the book value at the end of the planning horizon.

5 - The sum-of-years-digits method will be used for depreciation

6 - There is a 10% investment tax credit for the S/R's, the rack and for 50% of the controls (software is not eligible for tax credit).

For the rack supported building the following are assumed:

1 - The planning horizon is ten years.

2 - Land has no depreciation

3 - The S/R's, the rack, the controls, and the building have a ten year write-off period, and no salvage value

4 - The sum-of-years-digits method will be used for depreciation

5 - There is a 10% investment tax credit for the S/R's, the rack, the building and for 50% of the controls

It is also assumed that the after tax minimum attractive rate of return (MARR) and the applicable income tax rate will be provided by the user. The approach can be demonstrated by an example. Suppose the following are given:

ATMARR = 10%

Income Tax Rate = 50%

Land Cost = \$100,000

Building Cost (non-rack supported) = \$300,000

Building Cost (rack supported) = \$250,000

Rack Cost = \$300,000

S/R Cost = \$450,000 (includes \$125,000 for controls)

Total Recurring Cost = \$18,000

Sum-of-years-digits $= \frac{n(n+1)}{2}$, for n = 10, SOYD = 55

and for n = 40, SOYD = 820. The annual depreciation figures for the building, rack and S/R's are tabulated in Table 2-4. (Building 1 denotes non-rack supported construction, likewise 2 denotes rack supported building.) From the table it is found that:

Table 2-4. The Annual Depreciation on the Building, Rack, and S/R's

Year	Bldg 1	Bldg 2	Rack	S/R	(P/F,i,n)	PW_{1n}	PW_{2n}
1	14,634.12	45,454.525	54,545.43	81,818.145	0.9091	137,271.99	165,290.83
2	14,268.27	40,909.075	49,090.89	73,636.335	0.8264	113,213.07	135,229.03
3	13,902.42	36,363.625	43,636.35	65,454.525	0.7513	92,404.858	109,279.95
4	13,536.57	31,818.175	38,181.81	57,272.715	0.6830	74,440.914	86,927.254
5	13,170.72	27,272.725	32,727.27	49,090.905	0.6209	58,978.604	67,734.539
6	12,804.87	22,727.25	27,272.7	40,909.05	0.5645	45,716.946	51,318.13
7	12,439.02	18,181.8	21,818.16	32,727.24	0.5132	34,376.404	37,323.599
8	12,073.17	13,636.35	16,363.62	24,545.43	0.4665	24,716.205	25,445.429
9	11,707.29	9,090.9	10,909.08	16,363.62	0.4241	16,531.413	15,421.802
10	11,341.44	4,545.45	5,454.54	8,181.81	0.3855	9,628.938	7,009.084

$\sum_{n=1}^{10} PW_{1n} = 607,279.30$ $\qquad$ $\sum_{n=1}^{10} PW_{2n} = 700,979.61$

$$\sum_{n=1}^{10} PW_{1n} = 607,279.30$$

and
$$\sum_{n=1}^{10} PW_{2n} = 700,979.61$$

Hence, the present worth of tax savings due to increased depreciation is:

for non-rack supported = (607,279.30)(0.50) = 303,639.65

for rack supported = (700,979.61)(0.50) = 350,489.80

The total investment tax credit will be:

for non-rack supported = 300,000 + (450,000-125,000) + (125,000x0.50) 10%

= \$68,750

for rack supported = 68,780 + (250,000) 10% = \$93,750

The book value of the non-rack supported building at the end of the tenth year will be:

$$BV_{10} = FC - \sum_{i=1}^{10} D_i = 300,000 - 129,877.89 = 170,122.11$$

Therefore the PW cost of the system (non-rack) will be:

$$PW_{NR} = 300,000 + 300,000 + 450,000 + 18,000 \times 0.50(P/A,10\%,10) + 100,000$$
$$(1- (P/F,10\%,10)) - 303,639.65 - 68,750 - 170,122.11(P/F,10\%,10)$$

$$PW_{NR} = \$728,774.33$$

The PW cost for the rack-supported case will be:

$$PW_R = 250{,}000 + 300{,}000 + 450{,}000 + 18{,}000 \times 0.50(P/A,10\%,10) + 100{,}000$$
$$(1 - (P/F,10\%,10)) - 350{,}489.80 - 93{,}750$$
$$PW_R = \$672{,}506.20$$

It should be noted that the investment tax credit is limited to a certain amount which depends on the present tax liability of the firm as a whole. Let y denote the tax liability before credit, then the investment tax credit cannot exceed 12,500 + 0.5y. Hence, the investment tax credit, W, will be:

$$W = \text{Max} \left\{ (\text{initial investment cost})\ 10\%;\ 12{,}500 + 0.5y \right\}$$

The computer program computes the 10% of the initial investment cost and finds the value of y which equates the above expression to the computed value. The value of y is printed on the output as "The tax liability should be greater than". If the user finds that the firm's tax liabilities are smaller than the required amount, the necessary adjustment shall be made on the PW cost of the system.

It should be noted that if the user finds that the assumptions are not met, corresponding changes in the program can be made. In any case, the initial cost of each cost element appears in the output, hence the user has the freedom of computing the system cost through any desired approach.

Summary

The initial cost of an AS/RS is composed of the land cost, the building cost, the rack cost, and the S/R cost. Recurring costs include the S/R maintenance cost, the building maintenance cost, and labor cost. The corresponding expressions to compute each of the above cost elements have been presented throughout the chapter. In addition, an example demonstrating the calculation of the present worth cost has been presented. The analysis of the above mentioned cost elements in terms of the system variables will be treated in Chapter V.

CHAPTER III

DETERMINING SPACE REQUIREMENTS

Introduction

Determining the appropriate space required to operate economically and efficiently is an important step in the design of storage systems. The entire design is affected by the number of pallets to be stored in the system. From the standpoint of designing AS/R systems it is especially important to achieve a balance between the storage and throughput capacity of the system.

This chapter aims to develop a closed form expression to predict the space requirement "relationship" between dedicated and randomized storage given the ordering policies for items carried in stock. The particular inventory system at hand operates under the $<R,r,T>$ policy. The results presented in this chapter are not claimed to be theoretical and/or generalized in nature. The purpose is to demonstrate a unique approach for determining the space requirements from the inventory records of the company.

Definition of the System

In this study it is assumed that economic ordering policies for each item group are already determined according to procurement, inventory holding, shortage and system operating costs. In other words, the r and R values for each item group are assumed to be pre-determined with T, the review period, being kept fixed at one week. From (8):

> It will be recalled that if a periodic review system uses an Rr operating doctrine, then if at a review time the inventory position (in the backorders case) or the quantity on-hand plus on-order (in the lost sales case) is less than or equal to r, a quantity is ordered which is sufficient to bring the inventory position or the quantity on-hand plus on-order up to R.

For this study it is assumed that shortages are not backordered, i.e. shortages are considered to be lost sales. Hence, the inventory level to be checked is the on-hand plus on-order; henceforth, this quantity will be referred to as the inventory position. The R and r values for each item are explicitly stated in the model according to its demand group. There are 100 items carried in stock. The demand rate of each item is assumed to be independently Poisson distributed. The first 15 items are high demand items with 6 having seasonal demand pattern; the corresponding average demand rate is 20 items/week. The following 25 items are classified as medium demand items with 10 having seasonal variations; the average demand rate is 12 items/week. The remaining (60 items) are low demand items with no seasonal variation and an average demand rate of 4 items/week. The seasonal variation in the demand rate is generated by a sine curve with different amplitudes assigned to every item. The lead time is assumed to be deterministic and the same for each item stored in the warehouse.

Another important assumption is that all items require equal space. At first, such an assumption may seem unrealistic but the model can also be used when it does not hold. For instance, if item i requires twice as much space as that of item j, then item i can be considered as two separate items having equal mean demand rates and space requirements equivalent to that of j. It is also assumed that replenishments are instantaneous. In other words, when an order arrives the on-hand inventory is instantly increased by the amount received and no retrievals are allowed before the entire order is entered.

One controversial issue in regard to space requirement has been the relation between dedicated and randomized storage methods. For reasons which will become clear in the following paragraph, randomized storage will almost always require less space than dedicated storage. On the other hand, it is possible to achieve higher throughput levels by using dedicated storage. Thus, the question of whether randomized or dedicated storage is "best" does not have a simple answer.

Let I_{tk} be the on-hand inventory level of item k at the beginning of week t, and M_k denote the maximum value taken by I_{tk}. Then by definition, the storage space required for randomized storage, IGT, will be:

$$IGT = \max_t \sum_{k=1}^{100} I_{tk}$$

where t denotes the time period.

The storage space required for dedicated storage, IDS, will be:

$$IDS = \sum_{k=1}^{100} M_k$$

In words, IGT is equal to the maximum level reached by the "aggregate" on-hand inventory, while IDS is found by adding individually the maximum on-hand inventory level reached by each item. It can be readily seen that, unless all the orders for all items arrive exactly at the same point in time, randomized storage will always require less space than dedicated storage.

Finding the maximum on-hand inventory level analytically is a simple matter for the basic EOQ model. In continuous review model with a probabilisitc demand, it may be possible to set up confidence intervals for the on-hand inventory. Also, assigning a certain probability to the maximum on-hand inventory level that may be reached will be possible. However, when shortages are not backordered and when a periodic review model is used, even when a Poisson process generates the demand, the analytical approach becomes much more undesirable (see (8)). Hence, using the simulation approach seems more suitable for this particular inventory system under question.

The Simulation Model

The model proposed to simulate the system described in the previous section will be presented. The purpose for keeping track of certain variables of the system is motivated in the following sections. The variables of interest are taken into account after 52 weeks of simulation in order to ensure steady-state inventory levels. Each week is one iteration and the review period, T, is kept constant at one week.

Definition of the parameters:

DEM(i) = the mean demand rate of item i (Poisson distributed)

IAMP(i) = amplitude of the sine curve for item i (i = 1,2,...,16)

MC(i) = any integer number between 1 and 12; denotes the cycle length of the sine curve for item i.

RLAM(i) = any real number; denotes the origin of the sine curve for item i.

Hence, mean demand rate of item i = (DEM(i) + IAMP(i) $\sin\frac{2\pi}{MC(i)}$ (KL + RLAM(i)) . Where, IAMP(i) = 0 if item i does not have a seasonal variation.

IR(K) = re-order level for item k (r in the <R,r,T> notation).

IRT(K) = target inventory level for item k (R in the <R,r,t> notation).

ID(K) = is an array used to store weekly generated demand for each item.

K = denotes the item (K=1,...,100) (any integer number following k serves the same purpose).

KL = denotes the weeks (KL = 1,2,...,260)

LT = Lead time (deterministic)

Definition of the variables:

INV(K) = on-hand inventory level of item K

LS(K) = total number of lost sales for item k (is increased by the amount of shortage each time it occurs.)

ISAY = total number of lost sales summed over all items.

NOR(K,KL) = number of parts on the order placed in the KL^{th} week for item K.

MON(K) = total number of parts standing in the "on-order" status for item K (i.e. level of the on-order inventory of item K)

MCUM(K) = stores the total number of parts carried in inventory for item K (will be divided by number of weeks to find the average on-hand inventory).

MAX(K) = keeps track of the maximum on-hand inventory level reached by item K.

IGIT = stores the up-to-date maximum "aggregate" on-hand inventory level reached by the system.

ITOT = used to compute the aggregate on-hand inventory in a given week.

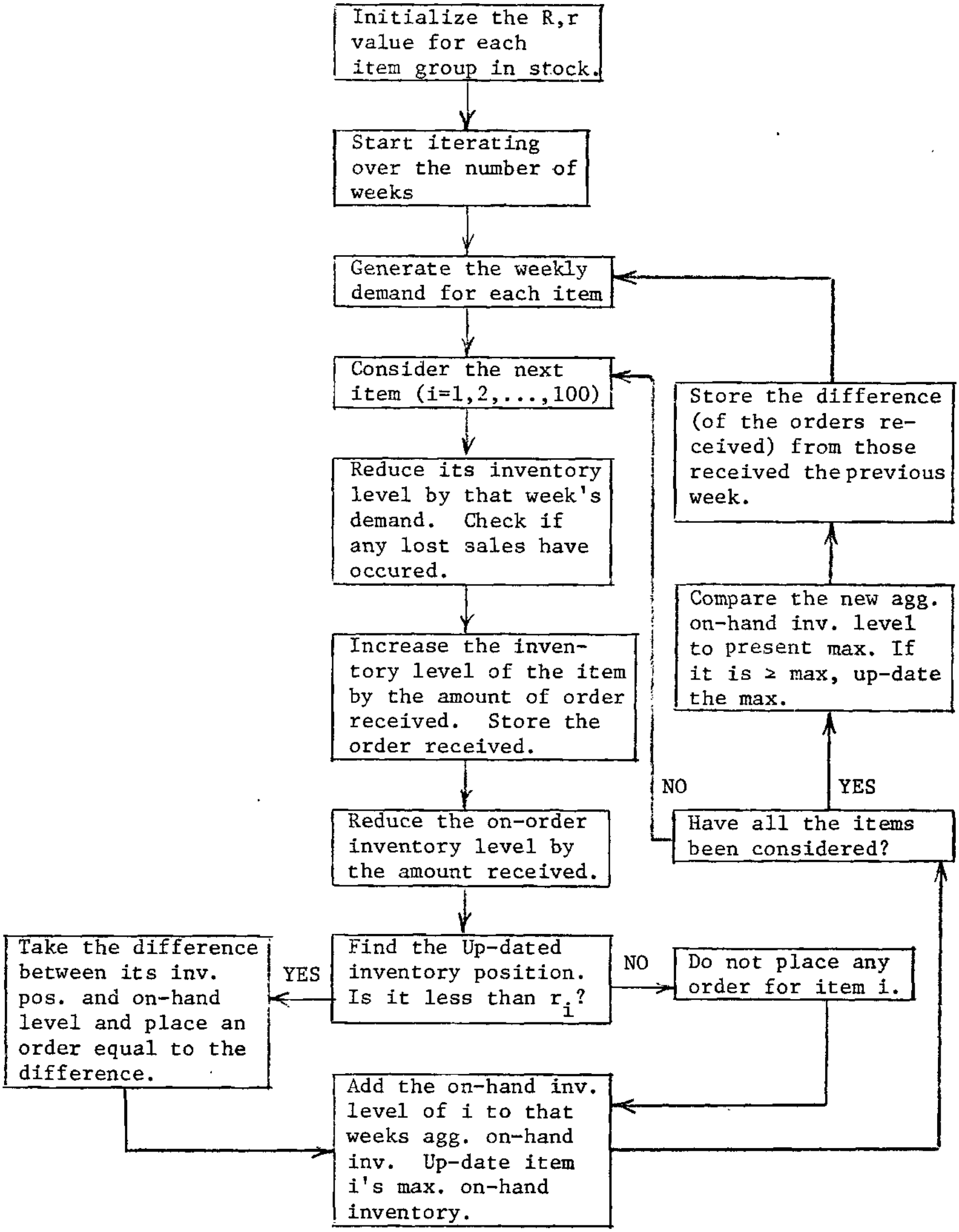
CUDF = stores (in a cumulative fashion) the difference between the total number of parts received in two consecutive weeks (will be divided by number of weeks to find the average difference)

RMX = keeps track of the maximum difference between the total number of parts received in two consecutive weeks.

A computer program was written to simulate the model defined by above parameters and variables. The flow-chart is presented in Figure 3-1. The program listing (written in Fortran IV) is given in Appendix 1. (The detailed flow-chart is given in Appendix 14).

Regression Analysis

Before the results obtained through regression analysis are presented, the reasons for selecting the independent variables will be motivated. The first independent variable is the average difference between the total number of parts received in two consecutive weeks (is equal to CUDF divided by the number of weeks). The purpose for selecting this variable can be explained in reference to Figure 3-2a. It is assumed that only two items are carried in stock. Item A has a demand of 6 parts/week and B has a demand of 5 parts/week (both demands are deterministic). Cycle length for both items equals two weeks. On the left part of the figure (i through iii), both of the orders are received on the same day. Thus, the total on-hand inventory level fluctuates between 0 and 22. At the very beginning of week 1, a total of 12 + 10 = 22 parts are received. At the beginning of week 2 however, 0 parts are received. The difference is 22 - 0 = 22. Similarly, the difference between weeks 2 and 3 is 22, as well. Hence, the average difference for this system will be 22 parts. For this case it will be



3-1. Flow-Chart of the Simulation Program

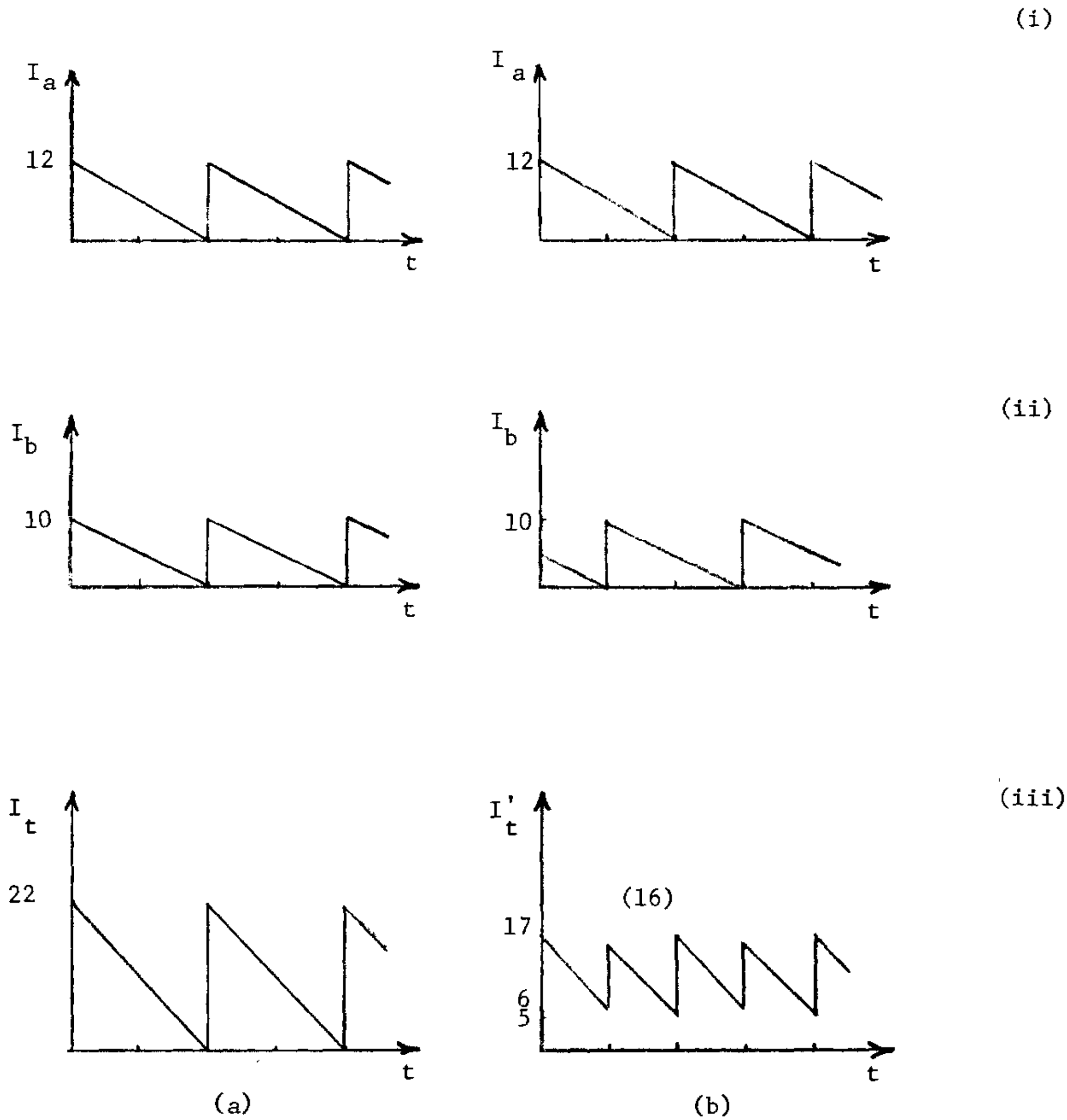


Figure 3-2. The effect of the difference between total number of parts received (in two consecutive weeks) on the relative space requirement.

noticed that both randomized and dedicated storage methods require 22 openings. For dedicated storage this figure is obtained by adding the maximum on-hand inventory levels for both items (shown in i and ii). For randomized storage on the other hand, it is obtained by looking at the maximum of the aggregate on-hand inventory level (shown in iii). Figure 3-2b portrays a different configuration with the cycle of item B shifted by one week. At the beginning of week 1 an order of 12 parts is received. In the following week another order of 10 parts is received, yielding a difference of 2 parts. Likewise, the difference between weeks 2 and 3 is 2 parts as well. Thus, the average difference for this system is 2 parts and there is a difference between the space requirements of the two storage methods. Dedicated storage requires 22 openings, again, while randomized storage requires 17 openings. The same logic will hold for any inventory system where n items are carried. Hence, the number of openings required by the two storage methods is affected by the difference between total number of items received in consecutive time periods.

The second independent variable is the "total number of lost sales". The relation between this second variable and storage space requirements of the two methods is not as obvious as for the first variable. However, it is true that the required storage space will be affected by the r and R values that have been chosen for each item. But this effect, in fact, is the result of the "combined" effect of the demand pattern and the <r,R> values. In this model there are only two variables which reflect this combined effect. They are the total lost sales and the average on-hand inventory of each item. Also, note that lost sales and on-hand inventory are inversely proportional. Predicting the direction of the above mentioned effect on the "relative" space requirements of the two methods is not straightforward. This point will be discussed in the last section of the present chapter. The variable of interest has been selected as the "total" lost sales because all items are assumed to require the same space.

The third and last independent variable was taken to be the maximum of the differences defined for the first variable. There is no readily observable reason for including this variable, but intuitively it is true that a very sharp change in the difference under question will not affect the average too much (because the system is simulated for 5 years, i.e. 260 weeks); on the other hand, it will have a dramatic effect on storage requirements (the system is assumed to be capable of storing any order size). Obviously, it can be argued that this variable may not be significant if such sharp changes are not expected. Let:

x_1' - average pre-defined difference (first independent variable)

x_2' - total number of lost sales (second independent variable)

x_3' - the maximum of the pre-defined differences (third independent variable)

y - the ratio of the space requirement of dedicated storage to that of randomized (the dependent variable)

Eleven simulation runs were made with each run having different <r,R> values and lead-times. The results are shown in Table 3-1. The related outputs can be found in Appendix 2. Since we have few independent

TABLE 3-1 RESULTS OF THE SIMULATION RUNS

X1	X2	X3	X4	X5	Y
62.7260	7.6914	229	52441	26.836	1.4270
61.0144	5.6041	212	44944	27.862	1.4189
65.9183	10.1478	252	53424	24.814	1.4753
62.1010	2.6207	223	49729	30.1053	1.3331
76.2115	4.1793	239	57121	28.3083	1.3472
90.2500	7.0406	359	128881	26.4136	1.3871
92.7644	19.4226	329	108241	19.0091	1.6233
87.6923	1.000	331	109561	34.7995	1.2109
104.4375	2.5713	369	136161	31.6281	1.2460
117.7308	9.9023	443	196249	26.1584	1.2900
126.1971	14.4369	411	168921	22.0214	1.3713

variables the "all possible regressions" technique seems to be appropriate. The result is presented in Table 3-2, where

$x_1 = x_1'$

$x_2 = (x_2')^{1/3}$

$x_3 = x_3'$

$x_4 = x_3^2$

x_5 = average aggregate on-hand inventory

and p = number of variables in the model including the intercept

R_p^2 = multiple correlation coefficient

$SS_R(p)$ = sum of squares due to regression

$SS_E(p)$ = sum of squares due to error

$MS_E(p)$ = mean square of error

$\bar{R}_p^2$ = adjusted multiple correlation coefficient

$$\bar{R}_p^2 = 1 - \frac{n-1}{n-p}(1-R_p^2), \text{ where } n = \text{number of observations}$$

C_p = Mallows statistic (a measure of bias and variance)

$$\hat{C}_p = \frac{SS_E(p)}{\hat{\sigma}^2} - n + 2p$$

In general, there are three criteria in selecting the best regression equation:

1 - MS_E should be minimized

2 - $\bar{R}_p^2$ should be maximized

3 - C_p should be less than or equal to p.

TABLE 3-2 ALL POSSIBLE REGRESSIONS

NUMBER OF VARIABLES	P	VARIABLES	R(P)**2	SSR(P)	SSE(P)	MSE(P)	$\overline{R}$(P)**2	C(P)
1	2	X1	<0.50					
1	2	X2	0.6045	0.0784	0.0513	0.0057	0.5606	56.63
1	2	X3	<0.50					
1	2	X4	<0.50					
1	2	X5	0.6976	0.0905	0.0390	0.0044	0.6640	41.33
2	3	X1,X2	0.9526	0.1237	0.0062	0.0008	0.9408	2.63
2	3	X1,X3	<0.50					
2	3	X1,X4	<0.50					
2	3	X1,X5	0.9297	0.1207	0.0091	0.0011	0.9120	6.302
2	3	X2,X3	0.9237	0.1199	0.0099	0.0012	0.9050	7.269
2	3	X2,X4	0.9365	0.1216	0.0082	0.0010	0.9206	5.212
2	3	X2,X5	0.7077	0.0918	0.0379	0.0047	0.6349	41.97
2	3	X3,X4	<0.50					
2	3	X3,X5	0.8959	0.1163	0.0135	0.0017	0.8706	11.73
2	3	X4,X5	0.9139	0.1186	0.0112	0.0014	0.8929	8.880
3	4	X1,X2,X3	0.9526	0.1237	0.0062	0.0009	0.9322	4.63
3	4	X1,X2,X4	0.9537	0.1238	0.0060	0.0009	0.9339	4.440
3	4	X1,X2,X5	0.9575	0.1243	0.0055	0.0008	0.9392	3.840
3	4	X1,X3,X4	<0.50					
3	4	X1,X3,X5	0.9338	0.1212	0.0086	0.0012	0.9054	7.650
3	4	X1,X4,X5	0.9297	0.1207	0.0091	0.0013	0.8996	8.310
3	4	X2,X3,X4	0.9401	0.1240	0.0078	0.0011	0.9145	6.630
3	4	X2,X3,X5	0.9258	0.1202	0.0096	0.0014	0.8944	8.927
3	4	X2,X4,X5	0.9403	0.1221	0.0075	0.0011	0.9145	6.330
3	4	X3,X4,X5	0.9346	0.1213	0.0085	0.0012	0.9068	7.510
4	5	X1,X2,X3, X4	0.9592	0.1245	0.0053	0.0009	0.9319	5.569
4	5	X1,X2,X3, X5	0.9579	0.1246	0.0055	0.0009	0.9298	5.766
4	5	X1,X2,X4, X5	0.9579	0.1244	0.0055	0.0009	0.9300	5.766
4	5	X1,X3,X4, X5	0.9557	0.1241	0.0058	0.0010	0.9261	6.130
4	5	X2,X3,X4, X5	0.9481	0.1231	0.0067	0.0011	0.9140	7.350
5	6	X1,X2,X3, X4,X5,X6	0.9689	0.1258	0.0040	0.0008	0.9378	6.000

Usually, it will not be possible to find one regression line that satisfies all three criteria simultaneously. However, inspection of Table 3-2 reveals the fact that the line including x_1, x_2 is the best among all possible regression lines. Furthermore, note that the second best line which includes x_1, x_2 and x_5 also provides a good fit. The regression line obtained by including x_1 and x_2 is (see Appendix 3):

$$y = 1.487 - (3.153 \times 10^{-3})\, x_1 + (2.079 \times 10^{-2}) x_2 \qquad (3\text{-}1)$$

The regression obtained by including x_1, x_2 and x_5 is (see Appendix 3):

$$y = 1.723 - (2.971 \times 10^{-3})\, x_1 + (1.478 \times 10^{-2})\, x_2 - (7.589 \times 10^{-3}) x_5 \qquad (3\text{-}2)$$

Note that use of (3-2) requires information on an additional independent variable, namely x_5. Also, from Table 3-2 it is seen that a third best line does not exist. (Table 3-2 has been constructed from the computer printout presented in Appendix 3).

Conclusions

One remark is concerned with the selection of the first independent variable, i.e. the average difference between the total number of "parts" received in two consecutive weeks. A system which uses a fixed order size policy can well use the total number of "orders" instead of parts.

Another remark is on the relation between lost sales and y. Recall that the regression coefficient of a particular independent variable will depend on other independent variables already in the model. However, it is instructive to note that both in 3-1 and 3-2, which are the only two lines that provide a good fit, x_2 has a positive coefficient. That is, the relative space required by dedicated storage increases as the total number of lost sales increases (the increase in storage space is at a decreasing rate). The reason underlying the above relation can be examined as follows; by definition

$$\text{Turnover Rate} = \text{TOR} = \frac{\text{average annual sales}}{\text{average on-hand inventory}}$$

But (lost sales) + (sales) = (demand). Therefore, (sales) = (demand) - (lost sales). Hence:

$$\text{TOR} = \frac{\text{average demand} - \text{average lost sales}}{\text{average on-hand inventory}}$$

Therefore,

$$\text{average lost sales} = \text{average demand} - (\text{TOR})(\text{av. on-hand inv.}) \quad (3\text{-}3)$$

In (3-3) as TOR decreases, lost sales increases. But from the regression line as lost sales increases, the relative space requirement of dedicated storage will increase. Hence, as TOR decreases the relative space requirement of dedicated storage increases. Likewise, as TOR increases, the so-called space requirement decreases. Therefore, in the light of the regression line and the above definition for TOR, it can be concluded that TOR and the relative space required by dedicated storage are inversely proportional, which is true. As stated by (13):

> This method (dedicated storage) is probably advantageous if each item is close to its maximum inventory on most days or if there is a high turnover rate.

The result of this study indicates that the word "probably" in the above statement can be dropped.

Note that the regression line only predicts the "relative" space requirements of dedicated and randomized storage methods. Mathematically,

> space required under dedicated storage = y x space required under randomized storage

$$\text{i.e.} \quad IDS = y \times IGT \qquad (3\text{-}4)$$

Hence, in order to compute the required storage space under one method, one needs to estimate the required storage space under the other method. Recall, the definitions of IDS and IGT as:

$$IGT = \max_t \sum_{k=1}^{100} I_{tk}$$

and

$$IDS = \sum_{k=1}^{100} M_k$$

From above expressions it is seen that estimating IDS is a trivial task, as long as the required data is provided. On the other hand, estimating IGT requires more computation and extensive data. Hence, in using the regression line for estimating space requirements one would first estimate IDS, and then use y to estimate IGT.

Summary

The relative space requirement under randomized and dedicated storage has been a controversial issue. This chapter has demonstrated a simulation approach for determining the relative space requirements of the two storage methods. The approach has been demonstrated within the framework of a hypothetical example. Simulation results show that $1.21 < y < 1.62$, i.e. dedicated storage may require up to 60% more space than randomized storage. Also, regression analysis shows that the relative space requirements of the two storage methods is best explained by the difference in total number of parts received in two consecutive weeks and the total lost sales.

CHAPTER IV

ANALYSIS OF THE STORAGE/RETRIEVAL MACHINE TRAVEL TIME

Introduction

The S/R travel time is the time required to perform a storage (or retrieval) for single command; and a storage and retrieval for dual command (definitions for single and dual command were given previously). The activities involved in a single command retrieve are:

1 - Leave the I/O point empty

2 - Travel to the pre-determined opening

3 - Retrieve the load

4 - Return to the I/O point and put-down the load

The activities involved in a single command storage are:

1 - Pick up the load at the I/O point

2 - Travel to the pre-determined opening

3 - Store the load

4 - Return empty to the I/O point

The activities involved in a dual command are:

1 - Pick up from the I/O point the load to be stored

2 - Travel to the first pre-determined opening

3 - Store the load

4 - Travel to the second pre-determined opening

5 - Retrieve the load

6 - Return to the I/O point and put down the load

Obviously, from a standpoint of increased throughput, it is always better to operate on a dual command; but since it requires both a storage and retrieval order simultaneously, this may not always be possible. Therefore, the system is assumed to operate on the dual command basis by a pre-determined percent of total operation numbers provided by the user (this parameter is referred to as "DUAL" in the program listing). The rest of the time the system operates on a single command basis. Hence, the expected travel time, ETH, will be:

$$\text{ETH} = (\text{DUAL} \times \text{DT}) + (1 - \text{DUAL})\text{ST} \quad \text{(in minutes)}$$

where DT = expected travel time for dual command

and ST = expected travel time for single command

The variance of expected travel time, VTT, will be:

$$\text{VTT} = \left[\text{DUAL} \times E(\text{DT}^2)\right] + \left[(1 - \text{DUAL}) \times E(\text{ST}^2)\right] - \text{ETH}^2$$

Also, VDT = variance of travel time for dual command.

and VST = variance of travel time for single command.

The travel time is a crucial variable of the system. The throughput capacity is directly determined by the expected travel time. Mathematically, throughput, TPUT, will be:

$$\text{TPUT} = \text{NOAI}/\text{ETH} \quad \text{(Operations/hr.)}$$

where $\frac{1}{ETH}$ (in operations/hr) is the throughput capacity of one aisle.

and NOAI = number of aisles.

The above expression for TPUT assumes that there will always be a storage and/or retrieval order present at the I/O point. Such a situation will occur when the storage/retrieval orders are scheduled in advance, so the arrival rate of the orders is deterministic. A typical example of this case is in-process storage where production schedules determine the storage/retrieval rates. On the other hand, if the orders arrive according to a probabilistic distribution, the actual throughput of the system has to be computed by:

$$TPUT = NOAI/(ETH + WT)$$

where WT = the time an order spends waiting in the queue.

The waiting time in the queue, WT, is found by assuming a $(M/G/1):(GD/\infty/\infty)$ queue. Hence, WT is given by:

$$WT = \frac{\lambda' \; V(t) + 1/\mu^2}{2(1-\eta)}$$

where

λ' = arrival rate per aisle = λ/NOAI

$V(t)$ = expected variance of travel time

μ = expected mean travel time

η = $\lambda'/\mu (\eta < 1)$

From the above expression given for TPUT, note that throughput will always decrease when WT is included, as long as WT $\neq$ 0. The amount of decrease in throughput will depend on the design under question. Specifically it will depend on the number of aisles because in determining λ', λ is divided by NOAI. The effect of including WT will be further discussed in Chapter VI.

Another remark is concerned with the estimation of λ. Recall that λ is the throughput level demanded from the system, expressed in operations/hr. By definition, an operation is either a storage or a retrieval. Hence, λ will be determined by adding the expected number of storages per hr. to the expected number of retrievals per hr. Furthermore, note that the magnitude of the aforementioned expected values should be equal in the long range. In addition, in cases where TPUT is only slightly greater than λ, the user should check the "peak" value of total operations/hr. against TPUT. If TPUT does not meet the peak value, then the program should be reexecuted with a higher value assigned to λ.

The program prints out both of the above mentioned throughput levels. Small variations in estimating the expected travel time may seem unimportant; but for a system with multiple aisles, this is not true. For example, underestimating the throughput of one aisle even by, say, three operations/hr. will lead to an underestimation of total throughput by 18 operations/hr. for a system with six aisles. In other words, the negative effect of slight variations in travel time estimation will grow seriously with the number of aisles. Hence, travel time estimation gains importance.

The Conventional Method of Computing Travel Time:

A method of computing S/R travel time has been developed by the AS/RS Product Section of the Material Handling Institute (9). It only holds for the randomized storage method. For single command travel time, it is found by adding the duration of the following operations (see Figure 4-1):

1. Retrieve a load at home station(I/O point).

2. Store the load at a storage location 1/2 the number of storage addresses along the aisle, and up 1/2 the number of vertical storage locations in that aisle. (In cases of simultaneous travel, use the longer of the travel or the lift times computed individually.)

3. Return to home position, empty and stop.

For dual command the total cycle time is found by adding:

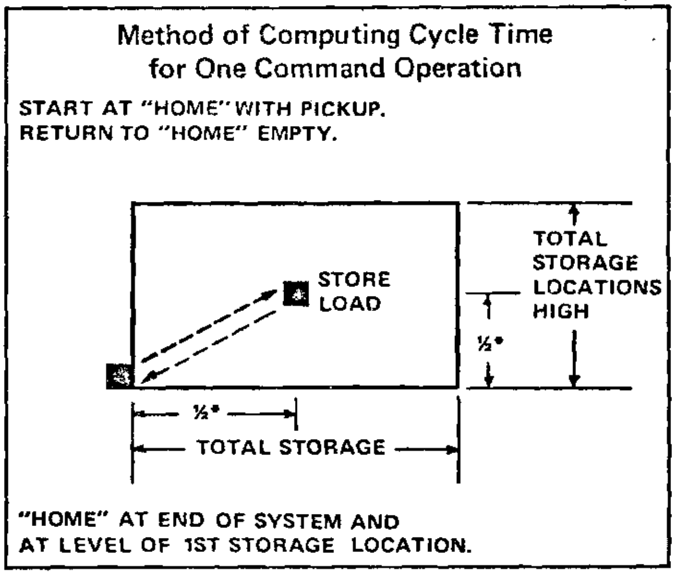
1. Pick up a load at home station.

2. Store the load at a storage location 1/2 the number of storage addresses along the aisle, and up 1/2 the number of vertical storage locations in that aisle. (For simultaneous travel use the longer of the two times).

3. Retrieve a load at a storage location 3/4 of the number of storage locations in that aisle, and up 3/4 the number of vertical storage locations in that aisle. (For simultaneous travel use the longer of the two times.)

4. Deposit load at home position.

Note: In cases where 1/2 or 3/4 of the addresses equals a fractional number, round off to the next higher (longer) address.



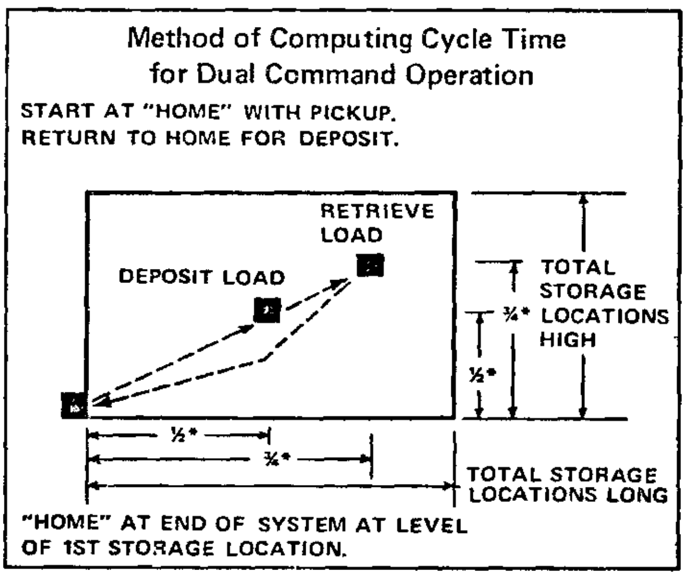


Figure 4-1. The Conventional Method of Computing S/R Travel Time for Single and Dual Command Trips (Reprinted from (9) with permission)

There are mainly two drawbacks of the above approach: First it is very unlikely that expected single command travel time will correspond to travel time to the opening at the center of gravity of the rack. The above statement is especially true if the rack is not square-in-time, and it can be supported by the following argument. Suppose in a given rack, vertical travel time is dominating, i.e. the travel time to the opening at the higher right-hand corner of the rack (across the I/O point) is equal to the vertical travel time to that opening (recall that travel time to an opening is determined by selecting the maximum of vertical and horizontal travel times to that opening). In this case, the openings at reasonably high levels will all have high (vertical) travel times compared to low (horizontal) travel times at lower levels of the rack. Hence, upon taking the average over all openings, we will definitely expect the center of travel time to be "above" the center of gravity of the rack. Likewise, for a rack in which horizontal travel time is dominating, one will expect the center of travel time to fall to the right of the center of gravity. Hence, based on the rack shape, the center of travel time will not coincide with the center of gravity. (The proof will be provided on an empirical basis in the next section.)

The second drawback is based on the fact that the third element in dual travel time computation will cause the dual command travel time to be over-estimated. Namely, given that the S/R has traveled to the first opening for storage (on an expected basis this will be travel time center), there is no reason for assuming that (on an expected basis) the second opening for retrieval will be at the indicated location (3/4 of the horizontal and vertical number of locations).

Randomized Storage

The expected travel time under randomized storage can be found accurately by complete enumeration. Let

H = number of levels (height)

L = number of columns (length)

N = total number of openings = $L \times H$

t_{oi} = travel time from origin to the i^{th} opening

t_{ij} = travel time from the i^{th} opening to the j^{th} opening.

The expected single command travel time, E(SC), can be found from

$$E(SC) = \frac{1}{N} \sum_{i=1}^{N} 2t_{oi} = \frac{2}{N} \sum_{i=1}^{N} t_{oi} \tag{4-1}$$

and its variance, V(SC), will be:

$$V(SC) = E(SC^2) - \left[E(SC)\right]^2$$

$$V(SC) = \frac{1}{N} \sum_{i=1}^{N} 4t_{oi}^2 - \left[E(SC)\right]^2$$

$$V(SC) = \frac{4}{N} \sum_{i=1}^{N} t_{oi}^2 - \frac{4}{N^2} \left(\sum_{i=1}^{N} t_{oi}\right)^2 \tag{4-2}$$

The expected dual command travel time, E(DC), can be found from

$$E(DC) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} (t_{oi} + t_{ij} + t_{jo}) \tag{4-3}$$

and its variance, V(DC), will be

$$V(DC) = E(DC^2) - \left[E(DC)\right]^2$$

$$V(DC) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} (t_{oi} + t_{ij} + t_{jo})^2 - \left[E(DC)\right]^2$$

$$V(DC) = \frac{2}{N^2-N} \sum_{i=1}^{N} \sum_{j=i+1}^{N} (t_{oi} + t_{ij} + t_{jo})^2 - \frac{4}{(N^2-N)^2} \times \left[\sum_{i=1}^{N} \sum_{j=i+1}^{N} (t_{oi} + t_{ij} + t_{jo})\right]^2 \tag{4-4}$$

Equations (4-1), (4-2), (4-3) and (4-4) have been used in a computer program that prints the true travel time and the travel time based on the conventional method. (The program listing is presented in Appendix 4.) The results are shown in Table 4-1. One of the drawbacks mentioned earlier for conventional single command travel time computation can be readily seen by observing columns E(SC) and conventional SC (con.sc.) in Table 4-1; it is seen that the conventional method always underestimates the true travel time. When columns E(DC) and con.DC. are observed, it can be seen that in some cases the conventional method underestimates the true travel time. It will be recalled that in mentioning

Table 4-1. Travel Times Obtained by Complete Enumeration and the Conventional Method

HV	VV	L	H	WTH	HEHT	E(SC)	V(SC)	E(DC)	V(DC)	CON.SC	CON.DC
220.0	40.0	10	5	40.0	48.0	0.5136	0.0708	0.6877	0.0472	0.5000	0.7500
230.0	50.0	20	7	42.0	46.0	0.5932	0.0651	0.7975	0.0473	0.5600	0.8400
230.0	50.0	40	12	42.0	46.0	1.0537	0.1797	1.4185	0.1354	0.9200	1.4000
240.0	60.0	40	10	40.0	40.0	0.7403	0.0682	0.9995	0.0539	0.5667	0.8417
240.0	40.0	50	10	40.0	40.0	1.0256	0.1398	1.3834	0.1091	0.8500	1.2500
220.0	45.0	50	10	40.0	40.0	0.9984	0.1243	1.3478	0.0983	0.7636	1.1364
260.0	50.0	53	10	48.0	40.0	0.9966	0.1342	1.3443	0.1047	0.8154	1.2231
200.0	45.0	48	12	44.0	44.0	1.2412	0.1965	1.6753	0.1548	0.9778	1.4667
205.0	52.0	30	7	48.0	40.0	0.6994	0.0695	0.9432	0.0534	0.5854	0.8780
225.0	56.0	35	8	44.0	44.0	0.7301	0.0674	0.9856	0.0530	0.5778	0.8622
260.0	60.0	30	7	33.0	32.0	0.4184	0.0217	0.5650	0.0171	0.3333	0.4872
260.0	60.0	15	3	36.0	41.0	0.2276	0.0062	0.3073	0.0047	0.2000	0.2846
260.0	60.0	20	4	31.0	36.0	0.2647	0.0086	0.3573	0.0066	0.2000	0.3000
250.0	35.0	30	8	51.0	51.0	1.0594	0.2220	1.4226	0.1603	0.9714	1.4857
295.0	35.0	30	8	51.0	51.0	1.0342	0.2390	1.3865	0.1689	0.9714	1.4857
295.0	35.0	29	8	51.0	51.0	1.0301	0.2423	1.3806	0.1706	0.9714	1.4857
295.0	35.0	28	8	51.0	51.0	1.0261	0.2456	1.3748	0.1723	0.9714	1.4857
295.0	35.0	27	8	51.0	51.0	1.0221	0.2490	1.3692	0.1740	0.9714	1.4857
295.0	35.0	25	8	51.0	51.0	1.0146	0.2558	1.3585	0.1774	0.9714	1.4857
200.0	62.0	40	9	60.0	45.0	1.0984	0.2335	1.4762	0.1707	1.0000	1.5000
200.0	62.0	40	7	60.0	47.0	1.0648	0.2564	1.4285	0.1823	1.0000	1.5000
240.0	70.0	19	4	50.0	52.0	0.3908	0.0220	0.5272	0.0166	0.3333	0.5000
240.0	70.0	18	4	50.0	52.0	0.3768	0.0195	0.5086	0.0148	0.3167	0.4750
240.0	85.0	13	4	50.0	52.0	0.2860	0.0103	0.3867	0.0078	0.2333	0.3417
240.0	85.0	11	3	50.0	51.0	0.2290	0.0072	0.3097	0.0053	0.1917	0.2917

the drawbacks of the conventional method, however, it was stated that the third element in the dual cycle computation will cause total cycle time to be over-estimated. The underestimated dual command times are due to the underestimation in single command. In other words, the second operation of a dual command is computed as if it is the second operation of a single command cycle and travel between two openings is added to this figure. Hence, if the underestimation in computing the second element overcomes the over-estimation in computing travel time between two openings, then the total dual cycle time will be underestimated as opposed to what was stated before. Therefore, in light of the above fact, it will be more correct to state the following for the conventional method: it will "always" underestimate single command cycle time and may underestimate or over-estimate dual cycle time.

Equations (4-1) through (4-4) are very useful in terms of providing a reliable answer. However, to use them in an algorithm, over and over again, will not be efficient. This is especially true for the double summation involved in computing dual cycle time. As N increases (roughly above 400 openings) the computation time for the double summation expression exceeds 60 cpu seconds (the default computation time for jobs executed through the terminal). Hence, from a computational viewpoint, it will be preferred to develop closed form expressions for the travel time computation. Furthermore, the existence of such expressions will prove to be very useful in establishing several facts concerning throughput (Chapter V).

One approach would be to represent the rack in a continuous manner instead of discretely spaced openings, and replace previous summations by integrations. In this case, the expected travel time for single command, $E(\overline{SC})$, will be:

$$E(\overline{SC}) = \int_{x=0}^{L} \int_{y=0}^{H} 2 \max \{\frac{y}{vv}, \frac{x}{hv}\} f(x) \cdot f(y) \, dy \, dx$$

$$E(\overline{SC}) = \int_{x=0}^{L} \int_{y=0}^{H} 2 \max \{\frac{y}{vv}, \frac{x}{hv}\} \frac{1}{L} \cdot \frac{1}{H} \, dy \, dx$$

where hv = horizontal travel velocity of the S/R

vv = vertical travel velocity of the S/R

We cannot integrate the above expression as long as the max operator is present. Hence, the rack will be partitioned as shown in Figure 4-2. (It is assumed that horizontal travel time is dominating). Hence,

$$L' = HR = H \frac{hv}{vv}$$

Therefore, $\frac{L'}{hv} = \frac{H}{vv}$ which means that the rack is square-in-time up to L'. Beyond L', horizontal travel time will be dominating. Since any point in region I lies above the diagonal, vertical travel time will be dominating in this region. Likewise, horizontal travel time will be dominating in region II. Subsequently, expected travel time to region I, T_I, will be:

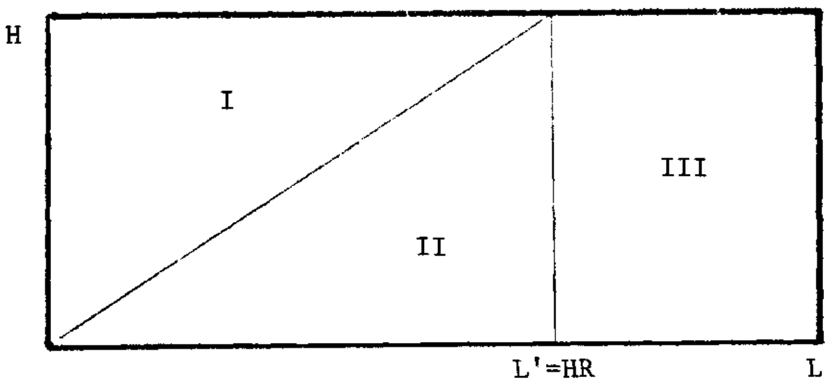


Figure 4-2. Partitioning of a rack where horizontal Travel time is dominating

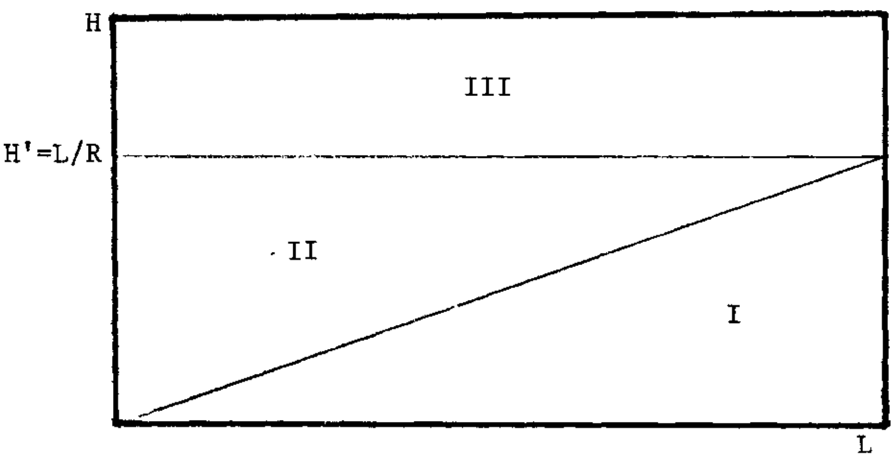


Figure 4-3. Partitioning of a rack where vertical travel time is dominating

$$T_I = \frac{L'}{L} \int_{x=0}^{HR} \int_{y=x/R}^{H} \frac{2y}{vv} \cdot \frac{1}{L'H}\, dy\, dx = \frac{2}{(vv)LH} \int_{x=0}^{HR} \frac{y^2}{2} \Big|_{x/R}^{H} dx$$

$$T_I = \frac{1}{(vv)LH} \int_{x=0}^{HR} H^2 - \frac{x^2}{R^2} = \frac{1}{(vv)LH} \left| H^2 x - \frac{x^3}{3R^2} \right|_0^{HR} = \frac{1}{(vv)LH} \left| H^3 R - \frac{H^3 R}{3} \right|$$

$$T_I = \frac{2H^2R}{3(vv)L}$$

Expected travel time to region II, T_{II}, will be:

$$T_{II} = \frac{L'}{L} \int_{x=0}^{HR} \int_{y=0}^{x/R} \frac{2x}{hv} \cdot \frac{1}{L'H}\, dy\, dx = \frac{2}{(hv)LH} \int_{x=0}^{HR} \frac{x^2}{R}\, dx = \frac{2}{(hv)LHR} \left| \frac{x^3}{3} \right|_0^{HR}$$

$$T_{II} = \frac{2}{(hv)HLR} \cdot \frac{H^3R^3}{3} = \frac{2H^2R^2}{3(hv)L}$$

If we substitute $(hv) = R(vv)$ in above expression for T_{II}, we will obtain

$$T_{II} = \frac{2H^2R}{3(vv)L} = T_I$$

The above equality is intuitively what one will expect because the rack formed by combining regions I and II is square-in-time. Expected travel time to region III, T_{III}, will be:

$$T_{III} = \frac{L-L'}{L} \int_{y=0}^{H} \int_{x=HR}^{L} \frac{2x}{hv} \cdot \frac{1}{L-L'} \cdot \frac{1}{H}\, dx\, dy = \frac{2}{(hv)LH} \int_{y=0}^{H} \frac{x^2}{2} \Big|_{HR}^{L}$$

$$T_{III} = \frac{1}{(hv)LH}(L^2H-H^3R^3) = \frac{L^2 - H^2R^2}{(hv)L}$$

Hence, the expected single command travel time, $E(\overline{SC})$, is:

$$E(\overline{SC}) = T_I + T_{II} + T_{III} = \frac{4H^2R}{3(vv)L} + \frac{L^2 - H^2R^2}{(hv)L} \qquad (4\text{-}5)$$

The variance of travel time, $V(\overline{SC})$, can be computed by first finding $E(\overline{SC}^2)$. In region I, it will be found by:

$$S_I = \frac{L'}{L} \int_{x=0}^{HR} \int_{y=x/R}^{H} \left(\frac{2y}{vv}\right)^2 \cdot \frac{1}{L'H}\, dy\, dx$$

Upon integration, one obtains the following expression for S_I:

$$S_I = \frac{RH^3}{(vv)^2L}$$

In region II, we will have:

$$S_{II} = \frac{L'}{L} \int_{x=0}^{HR} \int_{y=0}^{x/R} \left(\frac{2x}{hv}\right)^2 \cdot \frac{1}{L'H}\, dy\, dx$$

Upon integration, one obtains:

$$S_{II} = \frac{H^3R^3}{(hv)^2L}$$

If we substitute (hv) = R(vv), then

$$S_{II} = \frac{H^3R^3}{(vv)^2R^2L} = \frac{RH^3}{(vv)^2L} = S_I$$

In region III, we will have:

$$S_{III} = \frac{L-L'}{L} \int_{y=0}^{H} \int_{x=HR}^{L} \left(\frac{2x}{hv}\right)^2 \cdot \frac{1}{L-L'} \cdot \frac{1}{H} \, dx \, dy$$

Upon integration, one obtains

$$S_{III} = \frac{4(L^3 - H^3R^3)}{3(hv)^2L}$$

Hence

$$E(\overline{SC}^2) = S_I + S_{II} + S_{III} = \frac{2RH^3}{(vv)^2L} + \frac{4(L^3 - H^3R^3)}{3(hv)^2L} \tag{4-6a}$$

But

$$V(\overline{SC}) = E(\overline{SC}^2) - E^2(\overline{SC})$$

Thus,

$$V(\overline{SC}) = \frac{2RH^3}{(vv)^2L} + \frac{4(L^3 - H^3R^3)}{3(hv)^2L} - \left[\frac{4H^2R}{3(vv)L} + \frac{L^2 - H^2R^2}{(hv)L}\right]^2 \tag{4-6}$$

Similar results can be derived for the case where vertical travel time is dominating. The rack is partitioned as shown in Figure 4-3. We have $H' = \frac{L}{R} = \frac{L(vv)}{(hv)}$. Therefore, $\frac{L}{hv} = \frac{H'}{vv}$ which means that the rack is square-in-time up to H'. Beyond H' vertical travel time will be dominating. The dominating travel times for regions I and II are the horizontal and vertical times, respectively. In region I, the expected travel time, T_I, is:

$$T_I = \frac{H'}{H} \int_{x=0}^{L} \int_{y=0}^{x/R} \frac{2x}{hv} \cdot \frac{1}{H'L}\, dy\, dx$$

Upon integration we obtain

$$T_I = \frac{2L^2}{3(hv)HR}$$

In region II, the expected travel time, T_{II}, is:

$$T_{II} = \frac{H'}{H} \int_{x=0}^{L} \int_{y=x/R}^{L/R} \frac{2y}{vv} \cdot \frac{1}{H'L}\, dy\, dx$$

Upon integration we obtain

$$T_{II} = \frac{2L^2}{3(vv)HR^2}$$

Substituting $(vv) = (hv)/R$, then

$$T_{II} = \frac{2L^2}{3(hv)HR} = T_I$$

In region III, the expected travel time, T_{III}, is:

$$T_{III} = \frac{H-H'}{H} \int_{x=0}^{L} \int_{y=L/R}^{H} \frac{2y}{vv} \cdot \frac{1}{H-H'} \cdot \frac{1}{L}\, dy\, dx$$

After integrating we will obtain

$$T_{III} = \frac{H^2R^2-L^2}{H(vv).R^2}$$

Hence, the expected single command travel time (when vertical travel time is dominating) will be:

$$E(\overline{SC}) = T_I + T_{II} + T_{III} = \frac{4L^2}{3(hv)HR} + \frac{H^2R^2-L^2}{(vv)HR^2} \tag{4-7}$$

Development of the expression for $E(\overline{SC}^2)$ is very similar to that of equation (4-6). It is obtained from the following expression:

$$E(\overline{SC}^2) = \frac{H'}{H} \int_{x=0}^{L} \int_{y=0}^{x/R} \frac{4x^2}{(hv)^2} \cdot \frac{1}{H'L}\, dy\, dx + \frac{H'}{H} \int_{x=0}^{L} \int_{y=x/R}^{L/R} \frac{4y^2}{(vv)^2} \cdot \frac{1}{H'L}\, dy\, dx$$

$$+ \frac{H-H'}{H} \int_{x=0}^{L} \int_{y=L/R}^{H} \frac{4y^2}{(vv)^2} \cdot \frac{1}{H-H'} \cdot \frac{1}{L}\, dy\, dx$$

which, upon integration, reduces to

$$E(\overline{SC}^2) = \frac{2L^3}{(vv)^2 HR^3} + \frac{4(H^3R^3 - L^3)}{3(vv)^2 HR^3} = \frac{2L^3 + 4H^3R^3}{3(vv)^2 HR^3} . \tag{4-8a}$$

But

$$V(\overline{SC}) = E(\overline{SC}^2) - E^2(\overline{SC})$$

Thus,

$$V(\overline{SC}) = \frac{2L^3 + 4H^3R^3}{3(vv)^2 HR^3} - \left[\frac{4L^2}{3(hv)HR} + \frac{H^2R^2 - L^2}{(vv)HR^2}\right]^2 \tag{4-8}$$

The last case to be considered is the one where the rack is square-in-time. The expressions for this case can be developed by integration as done for the two previous cases. However, one can utilize the previous results from either one of the cases. Suppose the second case, where vertical travel time is dominating, is selected. Let $R' = \frac{H}{L}$. Then, since the rack is square-in-time, the following equations will hold

$$\frac{L}{hv} = \frac{H}{vv}$$

Therefore,

$$L(vv) = H(hv) \tag{4-9}$$

and

$$\frac{1}{R'} = R, \text{ i.e. } RR' = 1 \tag{4-10}$$

Now, consider equation (4-7)

$$E(\overline{SC}) = \frac{4L^2}{3(hv)HR} + \frac{H^2R^2 - L^2}{(vv)HR^2} = \frac{4L}{3(hv)R'R} + \frac{R'HR^2 - L}{(vv)R'R^2}$$

$$E(\overline{SC}) = \frac{4L}{3(hv)R'R} + \frac{R'HR^2 - L}{(hv)R'R}$$

Substituting equation (4-10) one obtains

$$E(\overline{SC}) = \frac{4L}{3(hv)} + \frac{HR - L}{(hv)} = \frac{L + 3HR}{3(hv)}$$

and using equation (4-9) we have

$$E(\overline{SC}) = \frac{4HR}{3(hv)} \tag{4-11}$$

for a rack that is square-in-time (the integration approach gives the same expression). For variance of travel time, equation (4-8a) will be modified as follows:

$$E(\overline{SC^2}) = \frac{2L^3 + 4H^3R^3}{3(vv)^2HR^3} = \frac{2L^2 + 4R'H^2R^3}{3(vv)^2R'R^3}$$

But, from (4-10) RR' = 1, therefore

$$E(\overline{SC^2}) = \frac{2L^2 + 4H^2R^2}{3(vv)^2R^2}$$

Substitute (4-9) for L to obtain

$$E(\overline{SC}^2) = \frac{2H^2R^2 + 4H^2R^2}{3(vv)^2R^2} = \frac{2H^2}{(vv)^2} \quad (4\text{-}12a)$$

Hence,

$$V(\overline{SC}) = \frac{2H^2}{(vv)^2} - \left[\frac{4HR}{3(hv)}\right]^2 \quad (4\text{-}12)$$

for a square-in-time rack.

Notice that in the first two cases the expressions for $E(\overline{SC})$ and $E(\overline{SC}^2)$ could have been further simplified by using $(vv) = R(hv)$. However, they were left as they are because the following transformation will be made. Let the longer side in travel time of the rack be 1.0. Then the shorter side in travel time will be b, where $0 < b \leq 1$. For example, in a rack where horizontal travel time is dominating, we have

$$b = \frac{\text{travel time to opening A}}{\text{travel time to opening B}}$$

where A is located at the upper left hand corner of the rack and B is located at the lower right hand corner of the rack. Consider now the first case where horizontal travel time was assumed to dominate. Mathematically, this means that

$$b = \frac{H/(vv)}{L/(hv)}$$

$E(\overline{SC})$ is given by equation (4-5) as

$$E(\overline{SC}) = \frac{4H^2R}{3(vv)L} + \frac{L^2 - H^2R^2}{(hv)L} = \frac{4H^2(hv)}{3(vv)^2L} - \frac{H^2(hv)}{(vv)^2L} + \frac{L}{hv}$$

$$E(\overline{SC}) = \frac{H^2(hv)}{3(vv)^2L} + \frac{L}{(hv)}$$

Recall that we standardize the rack by setting the larger travel time, in this case L/hv, to 1.0. Hence,

$$L/hv = 1.0 \text{ and } b = H/(vv) \tag{4-13}$$

Therefore,
$$E'(\overline{SC}) = \frac{b^2}{3} + 1 \tag{4-14a}$$

Now consider equation (4-6a):

$$E(\overline{SC^2}) = \frac{2RH^3}{(vv)^2L} + \frac{4L^3 - 4H^3R^3}{3(hv)^2L}$$

$$E(\overline{SC^2}) = \frac{2(hv)H^3}{(vv)^3L} + \frac{4L^2}{3(hv)^2} - \frac{4H^3(hv)}{3(vv)^3L} = \frac{2(hv)H^3}{3(vv)^3L} + \frac{4L^2}{3(hv)^2}$$

Using equation (4-13), we obtain

$$E'(\overline{SC^2}) = \frac{2}{3}b^3 + \frac{4}{3} \tag{4-15a}$$

Now consider the case where the vertical travel time is dominating. From equation (4-7), we have

$$E(\overline{SC}) = \frac{4L^2}{3(hv)HR} + \frac{H^2R^2 - L^2}{(vv)HR^2} = \frac{4L^2(vv)}{3(hv)^2H} + \frac{H}{(vv)} - \frac{L^2(vv)}{H(hv)^2}$$

Therefore, $$E(\overline{SC}) = \frac{L^2(vv)}{3(hv)^2H} + \frac{H}{(vv)}$$

But $$H/(vv) = 1.0 \text{ and } b = L/(hv) \tag{4-16}$$

Therefore, $$E'(\overline{SC}) = \frac{b^2}{3} + 1 \tag{4-14b}$$

For $E(\overline{SC^2})$, equation (4-8a) gives:

$$E(\overline{SC^2}) = \frac{2L^3}{(vv)^2HR^3} + \frac{4H^3R^3}{3(vv)^2HR^3} - \frac{4L^3}{3(vv)^2HR^3}$$

$$E(\overline{SC^2}) = \frac{2L^3(vv)}{(hv)^3H} + \frac{4H^2}{3(vv)^2} - \frac{4L^3(vv)}{3(hv)^3H} = \frac{2L^3(vv)}{3(hv)^3H} + \frac{4}{3}$$

Using equation (4-16) we obtain

$$E'(\overline{SC^2}) = \frac{2}{3}b^3 + \frac{4}{3} \tag{4-15b}$$

It will be readily seen that equations (4-14a) and (4-14b) are identical. So are equations (4-15a) and (4-15b). This result is not surprising because given the corresponding b (henceforth referred to as the "shape factor"), one would expect to obtain the same expressions for $E(\overline{SC})$ and $E(\overline{SC^2})$ in both types of racks. The answer for a square-in-time rack is now obvious. It can either be obtained from equations (4-11) and (4-12a) in a fashion similar to previous derivations or it will be obtained by simply setting b=1. From equation (4-11), we have:

$$E(\overline{SC}) = \frac{4HR}{3(hv)} = \frac{4H}{3(vv)}$$

Since the rack is square-in-time the above equation reduces to $E'(\overline{SC})=4/3$, which will also be obtained by letting b=1 in equations (4-14a) or (4-14b) (henceforth referred to as equation 4-14). The same value for $E'(\overline{SC}/b=1)$ is found by Graves et al. (5), who assumed a rack that is square-in-time. Furthermore, from equation (4-12a) we have:

$$E(\overline{SC^2}) = \frac{2H^2}{(vv)^2} \quad \text{but } H/(vv) = 1.0$$

$\therefore\ E'(\overline{SC^2}) = 2.0$ (again same result is obtained by setting b=1 in equation (4-15a) or (4-15b), henceforth referred to as equation (4-15)).

Next consider dual command travel time. For dual command the integration approach may not provide a solution because we can have travel "between" regions with different dominating times. Furthermore, even within a region where, say, horizontal travel time is dominating, the two locations may be so located that the vertical travel time will become dominant when travel between the two locations is considered. In addition, since we have two points (locations) to take into account, single command's double integration will be replaced by a quadruple integration. Hence, using an approach similar to that of single command does not seem to be promising. The following approach, however, makes it possible to develop an expression for the expected travel time between any two points. As before, let the longer side in travel time of the rack be 1.0 and let b be defined as the shape factor. Suppose we randomly select two points, x_1 and x_2 (see Figure 4-4). Let z denote the travel time between x_1 and x_2. Assuming that x_1 is fixed, the probability that travel time between the two points is less than or equal to z will be equal to the probability that x_2 lies anywhere in the square ABCD (we will initially assume that $z \leq b$). Define two sets: set I and set II. As seen from Figure 4-4, set I is the horizontal strip containing the square ABCD, while set II is the vertical strip containing the same square. Then, let us partially relax the assumption that x_1 is fixed and assume that it is now allowed to be anywhere on the dashed line labeled L_1, i.e. the square ABCD, in a sense, can now move anywhere within set I. Note that since $0 \leq x_1 \leq 1$, the left half of the square will fall outside the rack limits when $x_1 = 0$ (labeled A" and D" in the figure). Likewise, when $x_1 = 1$ the right half will fall out (labeled B' and C' in the figure). Hence, in finding the corresponding probability the above outliers will be taken into account as follows:

P (moving square coincides with ABCD) = area of square/area of set I

$P(x_1 = 0) = (\frac{1}{2}$ x area of square)/area of set I = $P(x_1 = 1)$.

$\therefore$ P(moving square coincides with ABCD; adjusted for $x_1 = 0$ "or" $x_1 = 1) \equiv P(H)$

$$= \frac{4z^2}{2z} - (\frac{2z^2}{2z}) = (2z - z^2)$$

Now let us further relax x_1 and assume that it can also be anywhere

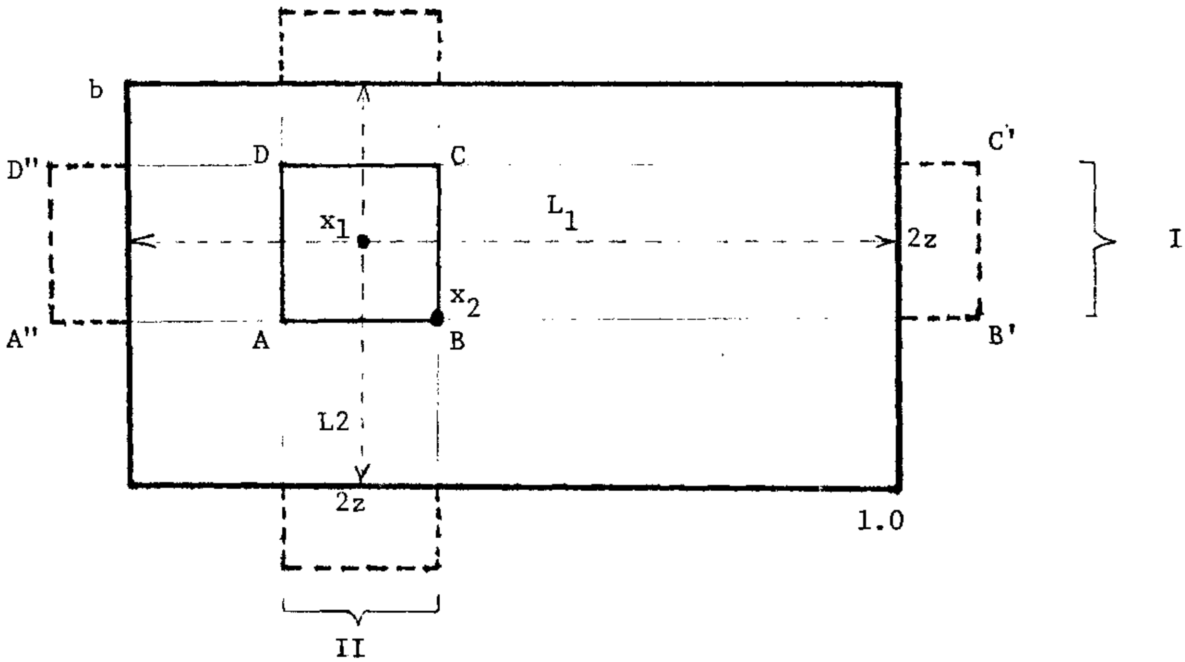


Figure 4-4. Location of the square ABCD

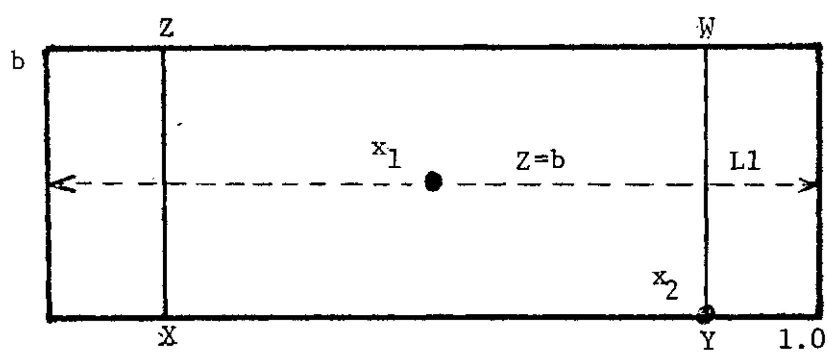


Figure 4-5. Location of the rectangle XYWZ

on the dashed line labeled L2, i.e. the square ABCD can also move within set II. This time since $0 \le x_1 \le b$, we need a similar adjustment for $x_1 = 0$ and $x_1 = b$. Thus,

P(moving square coincides with ABCD) = area of square/area of set II

$P(x_1=0)$ = (½ x area of square)/area of set II = $P(x_1=b)$

∴ P(moving square coincides with ABCD, adjusted for $x_1 = 0$ "or" $x_1 = b$) $\equiv P(V)$

$$= \frac{4z^2}{2zb} - \left(\frac{2z^2}{2zb}\right)^2 = \frac{2z}{b} - \frac{z^2}{b^2}$$

But, P(travel time between x_1 and $x_2 \le z$)= P(Having a square 2z x 2z)

$$= P(H) \,.\, P(V)$$
$$= (2z-z^2) \,.\, (2z/b - z^2/b^2)$$

Throughout the above argument it was assumed that $z \le b$. Consider now the case where $z \ge b$. As before, the two points x_1 and x_2 are randomly selected. However, for this case, since $z \ge b$ the previous square (denoted by ABCD) will become a rectangle (denoted by XYWZ) in Figure 3-5). With an argument similar to the case $z \le b$, it is true that

P(moving rectangle coincides with XYWZ) = area of rectangle/area of rack

$P(x_1 = 0)$ =(½ x area of rectangle)/area of rack = $P(x_1 = 1)$

∴ P(moving rectangle coincides with XYWZ; adjusted for $x_1 = 0$ "or" $x_1 = 1$) $\equiv$ P(H')

$$= \frac{2zb}{b} - \left(\frac{zb}{b}\right)^2 = 2z - z^2$$

Note that the rectangle XYWZ cannot be moved in the vertical direction. Mathematically, this means $P(V') = 1.0$, i.e. in the vertical direction the travel time is already less than or equal to z because $z \geq b$. Hence,

$$P(\text{travel time time between } x_1 \text{ and } x_2 \leq z) = P(H') \cdot P(V') = 2z - z^2.$$

The above result can be justified by setting $z=b$ in $P(V)$. Recall that

$$P(V) = 2z/b - z^2/b^2$$

if $z = b$, then $P(V) = 2 - 1 = 1.0$.

In summary, if $F(z)$ denotes the cumulative probability distribution of z, where z is the travel time between any two points, it was shown that

$$F(z) = \begin{cases} (2z - z^2)(2z/b - z^2/b^2) & \text{if } 0 < z \leq b \\ 2z - z^2 & \text{if } b < z \leq 1 \end{cases}$$

Therefore:

$$f(z) = \begin{cases} (2-2z)(2z/b - z^2/b^2) + (2z - z^2)(2/b - 2z/b^2) & \text{if } 0 < z \leq b \\ 2 - 2z & \text{if } b < z \leq b \end{cases}$$

$$\text{Hence, } E(z) = \int_0^1 z \cdot f(z)\, dz = \int_0^b zf(z)dz + \int_1^b zf(z)dz = E_1(z) + E_2(z)$$

$$E_1(z) = \int_0^b (2z - 2z^2)(2z/b - z^2/b^2) + (2z^2 - z^3)(2/b - 2z/b^2)dz$$

Upon opening the parentheses we obtain

$$E_1(z) = \int_0^b \frac{8z^2}{b} - \frac{6z^3}{b} - \frac{6z^3}{b^2} + \frac{4z^4}{b^2}\, dz = \frac{7}{6}\, b^2 - \frac{7}{10}\, b^3.$$

Also, $E_2(z) = \int_b^1 2z - 2z^2 dz = 1 - \frac{2}{3} - b^2 + \frac{2}{3}\, b^3$

Hence, $E(z) = \frac{1}{3} - b^2 + \frac{2}{3}\, b^3 + \frac{7}{6}\, b^2 - \frac{7}{10}\, b^3 = \frac{1}{3} + \frac{1}{6}\, b^2 - \frac{1}{30}\, b^3$ (4-17a)

The theoretical distribution of a complete dual cycle is difficult to determine. However, its expected value, $E'(\overline{DC})$, is

$$E'(\overline{DC}) = E'(\overline{SC}) + E(z)$$
$$= \frac{b^2}{3} + 1 + \frac{1}{3} + \frac{1}{6}\, b^2 - \frac{1}{30}\, b^3$$

$$E'(DC) = \frac{4}{3} + \frac{1}{2}\, b^2 - \frac{1}{30}\, b^3 \qquad (4\text{-}17)$$

It should also be noted that the E(z) when b=1, is:

$$E(z) = 1/3 + 1/6 - 1/30 = 7/15$$

The same value, i.e. 7/15 is found by Graves, et al (6) who studied a rack that is square-in-time.

The variance of dual command, $V'(\overline{DC})$, is found by the following analysis. First, a simulation program was written to estimate $E'(\overline{DC})$ and $V'(\overline{DC})$ for a given value of b. The program listing can be seen in

Appendix 5. The computer output for $0 \leq b \leq 1$, is presented in Table 4-2. From the aforementioned table one can graph the estimated coefficient of variation $\left[V'(\overline{DC})^{\frac{1}{2}} / E'(\overline{DC})\right]$ against b. From Figure 4-6 it can be seen that the relation is almost linear. The least squares estimates of the parameters produces the following model:

$$\text{Est. Coeff. of variation} = 0.3588 - 0.1321 \times b$$

$$(R^2 = 0.9705)$$

Also, note that the estimated values for $E'(\overline{DC})$ in Table 4-2 can be compared to those obtained from equation (4-17). Taking the percentage difference between the value of $E'(\overline{DC})$ obtained from (4-17) and the one taken from the table produces an average difference of 0.586% over the 11 runs.

In summary, for randomized storage the following are true:

$$E'(\overline{SC}) = \frac{b^2}{3} + 1 \quad \text{(from 4-14)}$$

$$E'(\overline{SC^2}) = \frac{2}{3}\, b^3 + \frac{4}{3} \quad \text{(from 4-15)}$$

$$E'(\overline{DC}) = \frac{4}{3} + \frac{1}{2}\, b^2 - \frac{1}{30}\, b^3 \quad \text{(from 4-17)}$$

$$V'(\overline{DC})^{\frac{1}{2}} = \left[0.3588 - 0.1321 \times b\right] E'(\overline{DC}) \quad (4\text{-}18)$$

where b is the shape factor and the rack is normalized (i.e. longer travel time is equal to 1.0). The formulae can be tested for a run taken from Table 4-1. Let us consider run number 12; we have:

Table 4-2. Simulation Results for Different b Values

SHAPE FACTOR= 0.00	MEAN DC T.TIME 1.3409	VAR. OF DC T.TIME .2206
SHAPE FACTOR= .10	MEAN DC T.TIME 1.3493	VAR. OF DC T.TIME .2233
SHAPE FACTOR= .20	MEAN DC T.TIME 1.3546	VAR. OF DC T.TIME .2234
SHAPE FACTOR= .30	MEAN DC T.TIME 1.4041	VAR. OF DC T.TIME .1853
SHAPE FACTOR= .40	MEAN DC T.TIME 1.4143	VAR. OF DC T.TIME .1922
SHAPE FACTOR= .50	MEAN DC T.TIME 1.4416	VAR. OF DC T.TIME .1801
SHAPE FACTOR= .60	MEAN DC T.TIME 1.5092	VAR. OF DC T.TIME .1675
SHAPE FACTOR= .70	MEAN DC T.TIME 1.5583	VAR. OF DC T.TIME .1709
SHAPE FACTOR= .80	MEAN DC T.TIME 1.6303	VAR. OF DC T.TIME .1735
SHAPE FACTOR= .90	MEAN DC T.TIME 1.7047	VAR. OF DC T.TIME .1663
SHAPE FACTOR= 1.00	MEAN DC T.TIME 1.7955	VAR. OF DC T.TIME .1688

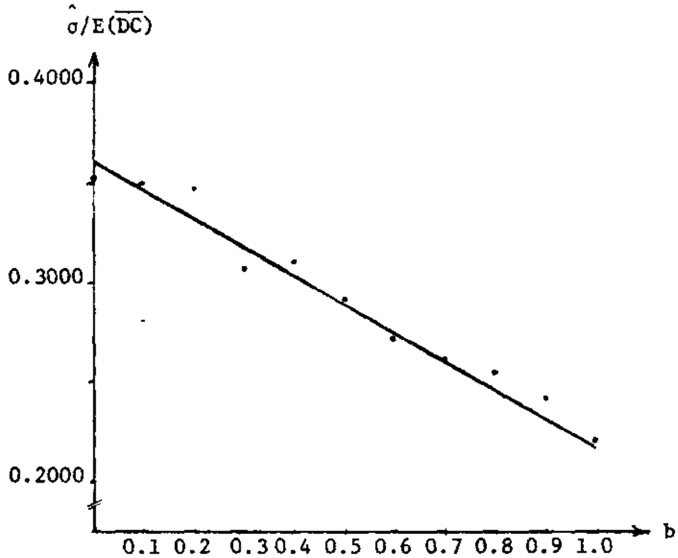


Figure 4-6. Scatter Diagram of Simulation Results

hv = 200 fpm vv = 45 fpm L = 48 H = 12 slot width = slot height = 44"

Therefore,

horizontal travel time = $\left[(44/12)48\right]/200 = 0.88$ min.

vertical travel time = $\left[(44/12)12\right]/45 = 0.9778$ min.

$\therefore\ b = 0.88/0.9778 = 0.900$

Hence, $E'(SC) = b^2/3 + 1 = 1.27$ min.

$E'(\overline{SC^2}) = 2b^3/3 + 4/3 = 1.8193$ min.

$E'(\overline{DC}) = 4/3 + b^2/2 - b^3/30 = 1.7140$ min.

$\therefore\ V'(\overline{SC}) = E(SC^2) - \left[E(SC)\right]^2 = 1.8193 - (1.27)^2 = 0.2064$

$V'(\overline{DC}) = \left\{\left[0.3588 - (0.1321)(0.90)\right]\ 1.714\right\}^2 = 0.1691$

Thus, we have the following results (numbers in parenthesis are the answers from Table 4-1).

$E(\overline{SC}) = 0.9778(1.27) = 1.2418\ (1.2412)$

$V(\overline{SC}) = (0.9778)^2(0.2064) = 0.1973\ (0.1965)$

$E(\overline{DC}) = (0.9778)(1.7140) = 1.6759\ (1.6753)$

$V(\overline{DC}) = (0.9778)^2(0.1691) = 0.1653\ (0.1548)$

From the above it can be noted that the expressions developed by assuming a continuous rack provide a good approximation to the true answer.

This will always be true if there are sufficiently high number of openings per rack. In practice, this number hardly falls under 30-40 openings.

Dedicated Storage

In dedicated storage particular slots are assigned to particular product classes, and within a set of slots assigned to one product class, storage and/or retrieval is performed randomly. One question which immediately arises from the above definition is to find a rule for assigning slots to product classes, so that the advantage gained by using dedicated storage (reduction in expected travel time) is maximized. One straightforward answer would be to assign high turn-over product classes (where turn-over is defined as operations/hr) to locations closer to the I/O point. In fact, throughout their study, Graves, et al (6,7) have used the above rule in assigning product classes to storage locations. In terms of minimizing expected single command travel time, such an assignment is not necessarily optimal. This point is proved in Francis and White (4), on the basis that it is not only the magnitude of the turnover rate that determines the priority given to a particular product class, but the number of openings required by that class should also be considered. Hence, if we let

C_i = number of operations/hr corresponding to product class i,

and A_i = number of openings required by product class i,

then the product class with the highest priority will be the one with the highest C_i/A_i ratio. (The inverse of this ratio is known as the

Cube Per Order, CPO index). Thus, one has to find the C/A ratio for every product class and then assign the closest locations to the highest C/A ratio class, the remaining closest locations to the second highest C/A ratio class and so forth, until all product classes are assigned to a particular set of openings. Suppose we had three product classes with decreasing C/A ratios. Then according to the value of A_I and A_{II}, the class boundaries will look either as in Figure 4-7a, 4-7b or 4-7c when the above assignment rule is used. In terms of dual command, however, we are not assured that such an assignment rule will minimize travel time because we have "travel between" two points (the fourth element of dual command operations). For a rack square-in-time, Graves, et al. (7) states the following:

> The exact shapes of the optimal boundaries are quite difficult to specify, although they are symmetric about the line from the I/O point to the opposite corner of the rack (due to square-in-time crane travel). "Square-L" boundaries possess this symmetry property. An alternate boundary configuration, also possessing square class boundaries and the symmetry property, has class I centered in the rack with class II and III boundaries forming concentric squares around it. This boundary pattern can be shown to minimize expected interleave (travel between) time, but not expected one-way (single command) time. The optimal class boundary minimizes expected round-trip time.

The study continues with an investigation of expected travel time sensitivity to class boundary shape. Their empirical findings are (1) expected travel between time is fairly insensitive to boundary shape, and (2) expected travel between time contributes "approximately" one-third to expected round-trip time (note that same result is

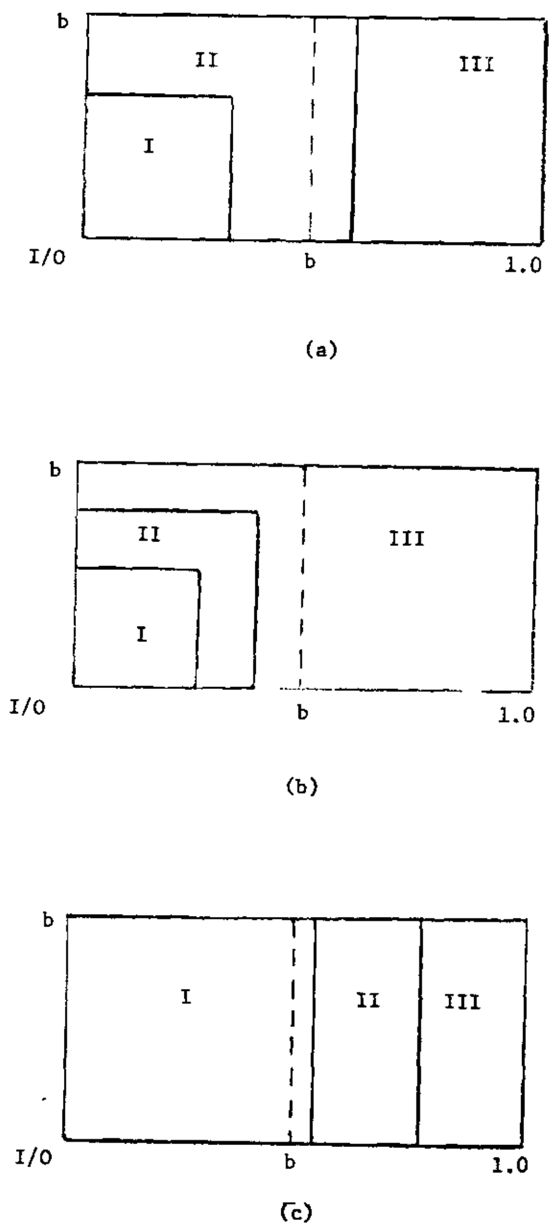


Figure 4-7. Alternative Configurations of a Rack with Three Product Classes

obtained by letting b=1 in equation 4-17a). A lower bound on expected travel time is twice the minimum expected single command time (achieved by square-L boundaries) plus the minimum expected travel between time (achieved by concentric-square boudaries). Their findings point out that square-L boundaries will result (at most) in an expected travel time only 3% above the aforementioned lower bound.

Based on the above mentioned results, this study has assumed "square-L" boundaries as those shown in Figure 4-7b; i.e. product classes are allocated to openings starting from those close to the I/O. Now, let k_{ij} denote the travel time to the j^{th} opening assigned to product class i. Suppose the k_{ij} values are k_{i1}, k_{i2}, . . ., k_{im} for some product class i requiring m slots. Then, if x denotes single command travel time, we have:

x	$p(x)$	x^2
$2k_1$	$1/m$	$4k_1^2$
$2k_2$	$1/m$	$4k_2^2$
.	.	.
.	.	.
.	.	.
$2k_m$	$1/m$	$4k_m^2$

Hence,

$$E(x) = \sum_i E(x/\text{class i}) \cdot (\text{class i})$$

$$\therefore \quad E(x) = \sum_i \frac{2}{m_i} \sum_{j=1}^{m_i} k_{ij} \cdot \frac{c_i}{C} \qquad (4\text{-}19)$$

where $$C = \sum_i c_i$$

Also, $$E(x^2) = \sum_i E(x^2/\text{class } i) \cdot p(\text{class } i)$$

$$E(x^2) = \sum_i \left(\frac{4}{m_i} \sum_{j=1}^{m_i} k_{ij}^2\right)\frac{c_i}{c} \tag{4-20}$$

Using (4-19) for $E(x)$ and (4-20) for $E(x^2)$ we have:

$$V(x) = E(x^2) - E^2(x) = \frac{4}{C} \sum_i \frac{c_i}{m_i} \sum_{j=1}^{m_i} k_{ij}^2 - \frac{4}{C^2}\left[\sum_i \frac{c_i}{m_i} \sum_{j=1}^{m_i} k_{ij}\right]^2$$

Next consider the dual command travel time. The probability of a dual trip that involves slot i and slot j is:

$$P(i,j) = \frac{\tilde{c}_i}{m_i} \cdot \frac{\tilde{c}_j}{m_j} \quad \text{where } \tilde{c}_k = c_k/C$$

Since we are not allowed to visit the same opening twice, i.e. since we cannot have $i=j$, each $P(i,j)$ for $i \neq j$ has to be divided by

$$1 - (\text{probability that } i=j) \equiv 1 - \sum_i \frac{\tilde{c}_i^2}{m_i^2} = 1 - \delta$$

Hence, if Y denotes dual command travel time, we have

$$E(Y) = \sum_i \sum_{j \neq i} \frac{\tilde{c}_i \tilde{c}_j / m_i m_j}{1 - \delta} (t_{oi} + t_{ij} + t_{jo})$$

$$E(Y) = \sum_i \sum_{j \neq i} \frac{\tilde{c}_i \cdot \tilde{c}_j}{m_i m_j (1-\delta)} (t_{oi} + t_{ij} + t_{jo})$$

The above expression for E(Y) can be further simplified by noting that $t_{oi} + t_{ij} + t_{jo} = t_{oj} + t_{ji} + t_{io}$. Hence,

$$E(Y) = \sum_{i} \sum_{j>i} \frac{2\tilde{c}_i\tilde{c}_j}{m_i m_j(1 - \delta)} (t_{oi} + t_{ij} + t_{jo}) \qquad (4\text{-}22)$$

Also, $$E(Y^2) = \sum_{i} \sum_{j>i} \frac{2\tilde{c}_i\tilde{c}_j}{m_i m_j(1 - \delta)} (t_{oi} + t_{ij} + t_{jo})^2 \qquad (4\text{-}23)$$

Therefore, $V(Y) = E(Y^2)$, which is obtained from (4-22) and (4-23).

The double summation involved in randomized storage dual command is also present in the expressions for dedicated storage dual command travel time. Hence, we once again face the issue of the inefficiency involved in using (4-22) and (4-23) in a repetitive manner. Partitioning the rack in a fashion similar to Figures (4-2) and (4-3) is not feasible for this case because in addition to zones with different dominating velocities, we also have zones that correspond to different product classes. Furthermore, in terms of programming it is difficult to determine a general partitioning scheme based on product class zones because any particular configuration of the rack will depend upon the number of product classes and their corresponding m_i values, both of which are provided by the user. In addition, any analytical approach will be based on that particular configuration obtained at a particular instance of the problem. Hence, an analytical approach seems to be very inefficient if the number of product classes is not restricted to a certain value. However, fixing the number of product classes will lead to loss of generality. One solution to the above problem is to simulate the rack after allocating the classes to the openings. Another solution would be to pull four unit loads together when total number of openings exceed a certain value, and then use (4-22) and (4-23) to evaluate E(Y) and $E(Y^2)$. Computational experience shows that, on the average, pulling four loads in to one provides a better approximation than simulation. However, when the the number of openings on the rack exceeds approximately 1600 openings (a situation which occurs in the Fibonacci search over the lower end of the uncertainty interval where number of aisles is a small number) the computation time again grows seriously. Hence, simulation has been used for approximating the the dual cycle travel time. (Single command travel time is automatically found by enumeration while allocating the product classes to the rack openings.) A sample of 12 runs has been analyzed in terms of checking the accuracy achieved by simulation.

The data related to the sample runs are presented in Table 4-3. Table 4-4 tabulates the results. The first two columns represent the true answers given by enumeration. The third and fourth columns represent simulation results obtained by a simulation time equal to 2*G, where G = total number of slots in the rack. The last two columns represent the results obtained by a simulation time equal to 10,000 trips. Table 4-5 tabulates the percent differences between the true answers and those obtained by simulation.

TABLE 4-3 SAMPLE RUNS DATA

NOPR	NOL	ICOL	1. C/A	2. C/A	3. C/A	4. C/A	5. C/A
2	5	10	50/25	25/25			
2	5	10	100/20	10/30			
3	7	15	50/30	30/30	30/45		
3	7	15	90/30	30/30	15/45		
3	7	15	30/30	30/30	45/45		
4	10	20	100/50	90/50	80/50	70/50	
4	10	20	100/50	85/50	84/50	70/50	
4	10	20	100/25	70/35	60/60	40/80	
4	10	20	100/50	98/50	96/50	94/50	
5	10	30	300/60	240/60	180/60	120/60	60/60
5	10	40	200/40	180/60	150/80	110/100	60/120
5	10	40	164/80	160/80	156/80	152/80	148/80

TABLE 4-4. ENUMERATION VS. SIMULATION RESULTS

1. E(DC) (TRUE)	2. V(DC) (TRUE)	3. E(DC) 1.SIMUL.	4. V(DC) 1.SIMUL.	5. E(DC) 2.SIMUL.	6. V(DC) 2.SIMUL.
0.5224	0.0303	0.4646	0.0226	0.4797	0.0262
0.3710	0.0198	0.3465	0.0140	0.3466	0.0154
0.7182	0.0630	0.6932	0.0487	0.6767	0.0558
0.5748	0.0507	0.5852	0.0373	0.5481	0.0433
0.8239	0.0537	0.7788	0.0486	0.7775	0.0496
1.1112	0.1204	1.0473	0.1087	1.0662	0.1112
1.1157	0.1190	1.0517	0.1074	1.0711	0.1101
0.8490	0.1366	0.8076	0.1145	0.8121	0.1229
1.1554	0.1153	1.0887	0.1069	1.1079	0.1073
1.0742	0.0998	1.0466	0.0840	1.0506	0&0918
1.0933	0.1325	1.0800	0.1230	1.0680	0.1281
1.4101	0.1116	1.3614	0.1051	1.3707	0.1075

TABLE 4-5 PERCENT DIFFERENCES BETWEEN ENUMERATION AND SIMULATION RESULTS

PERCENT DEVIATION		PERCENT DEVIATION	
1. VS 3.	2. VS 4.	1. VS 5.	2. VS 6.
11.06	25.41	8.17	13.53
6.60	29.29	6.58	22.22
3.48	22.70	5.78	11.43
-1.81	26.43	4.65	14.60
5.47	9.50	5.63	7.64
5.75	9.72	4.05	7.64
5.74	9.75	4.00	7.48
4.88	16.18	4.35	10.03
5.77	7.29	4.11	6.94
2.57	15.83	2.20	8.02
1.22	7.17	2.31	3.32
3.45	5.82	2.79	3.67
4.82	15.42	4.55	9.71

It is seen that by increasing simulation time the average percent deviation reduces from 4.82% to 4.55% for mean travel time. For its variance, the average percent deviation reduces from 15.42% to 9.71% (a considerable reduction). Furthermore, from Table 4-5, note that high percent deviations are caused by widely spread C/A ratios. For the purpose of this study a simulation time of 10,000 trips is sufficient. However, in cases where the C/A ratios are widely spread and/or number of openings in the rack is a comparatively large number, the corresponding statement in the computer program should be modified. (The program listing from which the true answers were obtained is presented in Appendix 6. The listing of the program written to simulate trips is presented in Appendix 7. The computer printout from which Table 4-4 has been prepared, is presented in Appendix 8.)

Note that, in computing S/R travel time we have not considered the load handling times. Pick-up and put-down times are assumed to be constant and deterministic, i.e. they do not vary with the shape of the rack. Therefore, they are simply added to the expected mean travel time in order to find the time required to complete an entire cycle.

Statistical Distribution of Travel Times

Analyzing the travel time distribution is important in terms of highlighting future simulation studies. First consider randomized storage. Expressions for the mean and variance of single and dual command travel times have been presented earlier in the chapter. However, the theoretical distribution of travel time was only developed for travel between tow points. The theoretical distribution of single command travel time can be developed as follows; let,

$F(v)$ = the probability that the travel time to a point is less than or equal to v.

If the rack is normalized by setting the larger travel time equal to 1, then

$$F(v) = \begin{cases} 2v^2/b & \text{if } v \le b \\ 2v & \text{if } v > b \end{cases}$$

where b = the shape factor, and $0 < v \le 1$.

Note that when b=1, f(v) is a triangular distribution, and when b=0, f(v) becomes a uniform distribution. For $0 < b < 1$, f(v) will linearly increase up to 2b, and then will become an uniform distribution after 2b. Also, there will be a discontinuity at b. Using the above expression for F(v) to derive f(v), and evaluating $\int_0^1 v \cdot f(v)$ and $\int_0^1 v^2 \cdot f(v)$ produces the same expressions obtained earlier for $E'(\overline{SC})$ and $V'(\overline{SC})$, respectively. The program listing presented in Appendix 5 was used to plot the histogram of single command travel time. The result is presented in Figure 4-8. Note the shape of f(v) as predicted earlier.

The histogram for dual command travel time was also obtained from the above mentioned program. The result is presented in Figure 4-8. Theoretically, dual command travel time will be Beta distributed (it is the sum of the maximum of two uniform random variables). Plots shown in Figure 4-9 confirm this fact.

The theoretical distribution of single command travel time is difficult to develop when dedicated storage is used. The same is true

Figure 4-8. The Statistical Distribution of Randomized Storage Single Command Travel Time

```
REWIND,FB10

READY.
RUN

 78/11/01. 21.23.16.
PROGRAM   HISTOMF

INPUT DATA FILE NAME
? FB10
INPUT ENDPOINTS OF ENTIRE INTERVAL,A & B
? 0.05 2.10
INPUT UNIT LENGTH OF CLASS INTERVALS, U
? 0.14
INPUT TOTAL NO. OF DATA POINTS, N
? 900

SAMPLE MEAN= 1.33634
SAMPLE VARIANCE= .212721
STD. DEVIATION= .461216
MEDIAN= 1.41202
MODAL CLASS INTERVAL IS  1.87  TO  2.01
MODE= 1.80867
RANGE= 1.95974
MIDRANGE= 1.01941
MIDPOINT*FREQUENCY BASED MEAN= 1.33536
MIDPOINT*FREQUENCY BASED VARIANCE= .213343
STD. DEV.= .461891

        0   15   30   45   60   75   90  105  120
        ''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''
'''''''''''''''''''''''''''''''''''''''''''''''''''''
        '    '    '    '    '    '    '    '    '    '    '    '    '    '
    '    '    '    '    '    '    '    '    '    '    '
 .05    -
 .12    -*( 5 )
 .19    -
 .26    -****( 13 )
 .33    -
 .4     -*********( 29 )
 .47    -
 .54    -**********( 31 )
 .61    -
 .68    -***********( 35 )
 .75    -
 .82    -**********************( 67 )
 .89    -
 .96    -*****************( 52 )
1.03    -
1.1     -***************************( 81 )
1.17    -
1.24    -***********************( 69 )
1.31    -
1.38    -*******************************( 94 )
1.45    -
1.52    -*********************************( 100 )
1.59    -
1.66    -************************************( 108 )
1.73    -
1.8     -*********************************( 99 )
1.87    -
1.94    -***************************************( 117 )
2.01    -
2.08    -( 0 )
2.15    -
```

```
SAMPLE MEAN= 1.27592
SAMPLE VARIANCE= .200296
STD. DEVIATION= .447544
MEDIAN= 1.34283
MODAL CLASS INTERVAL IS  1.7  TO  1.81
MODE= 1.71833
RANGE= 1.95974
MIDRANGE= 1.01941
MIDPOINT*FREQUENCY BASED MEAN= 1.27507
MIDPOINT*FREQUENCY BASED VARIANCE= .201464
STD. DEV.= .448847

        0   15   30   45   60   75   90  105  120
        ''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''
'''''''''''''''''''''''''''''''''''''''''''''''''''''
        '    '    '    '    '    '    '    '    '    '    '    '    '    '
    '    '    '    '    '    '    '    '    '    '    '
 .05    -
 .105   -*( 5 )
 .16    -
 .215   -*****( 16 )
 .27    -
 .325   -*****( 17 )
 .38    -
 .435   -*******( 23 )
 .49    -
 .545   -********( 26 )
 .6     -
 .655   -************( 36 )
 .71    -
 .765   -*******************( 59 )
 .82    -
 .875   -*****************( 53 )
 .93    -
 .985   -***********************( 70 )
1.04    -
1.095   -*********************( 64 )
1.15    -
1.205   -***********************( 69 )
1.26    -
1.315   -***************************( 83 )
1.37    -
1.425   -******************************( 91 )
1.48    -
1.535   -*******************************( 94 )
1.59    -
1.645   -*******************************( 94 )
1.7     -
1.755   -***********************************( 105 )
1.81    -
1.865   -****************( 50 )
1.92    -
1.975   -***************( 45 )
2.03    -
2.085   -( 0 )
2.14    -

SRU      3.245 UNTS.

RUN COMPLETE.
```

```
PROGRAM   HISTOMF

INPUT DATA FILE NAME
? FB08
INPUT ENDPOINTS OF ENTIRE INTERVAL,A & B
? 0.05 2.2
INPUT UNIT LENGTH OF CLASS INTERVALS, U
? 0.13
INPUT TOTAL NO. OF DATA POINTS, N
? 1000

SAMPLE MEAN= 1.22172
SAMPLE VARIANCE= .199719
STD. DEVIATION= .4469
MEDIAN= 1.2689
MODAL CLASS INTERVAL IS  1.48  TO  1.61
MODE= 1.48848
RANGE= 1.95974
MIDRANGE= 1.01941
MIDPOINT*FREQUENCY BASED MEAN= 1.22169
MIDPOINT*FREQUENCY BASED VARIANCE= .199042
STD. DEV.= .446141

        0    15    30    45    60    75    90   105   120

 .05    -
 .115   -**( 7 )
 .18    -
 .245   -******( 20 )
 .31    -
 .375   -**********( 30 )
 .44    -
 .505   -************( 35 )
 .57    -
 .635   -*****************( 53 )
 .7     -
 .765   -*********************( 64 )
 .83    -
 .895   -***************************( 81 )
 .96    -
 1.025  -***********************( 70 )
 1.09   -
 1.155  -*********************************( 101 )
 1.22   -
 1.285  -***********************************( 105 )
 1.35   -
 1.415  -*************************************( 113 )
 1.48   -
 1.545  -**************************************( 116 )
 1.61   -
 1.675  -************************( 73 )
 1.74   -
 1.805  -*********************( 63 )
 1.87   -
 1.935  -***********************( 69 )
 2.     -
 2.065  -( 0 )
```

```
SAMPLE MEAN= 1.17391
SAMPLE VARIANCE= .208242
STD. DEVIATION= .456335
MEDIAN= 1.10439
MODAL CLASS INTERVAL IS  1.09  TO  1.22
MODE= 1.19065
RANGE= 1.95974
MIDRANGE= 1.01941
MIDPOINT*FREQUENCY BASED MEAN= 1.17398
MIDPOINT*FREQUENCY BASED VARIANCE= .207176
STD. DEV.= .455166

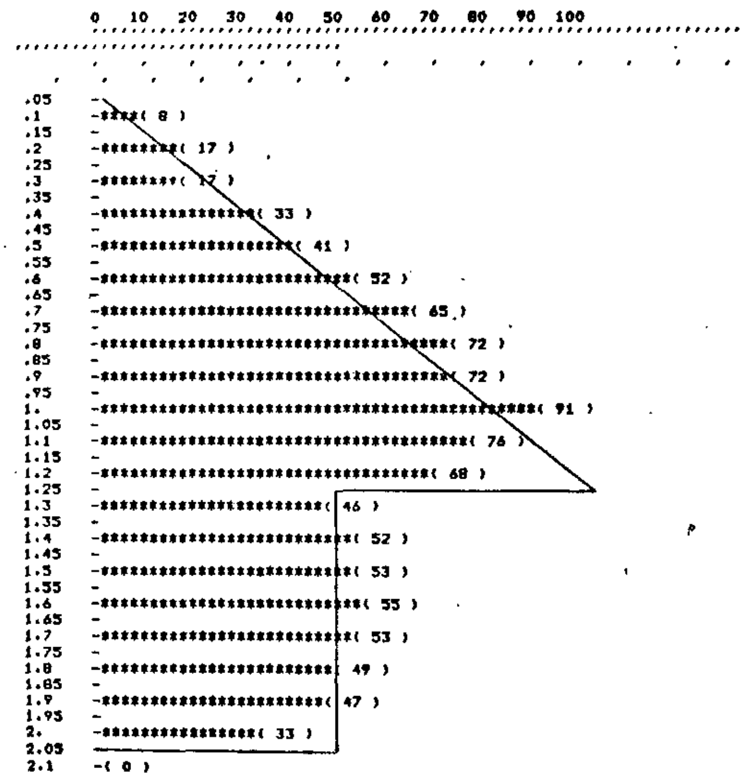
        0    15    30    45    60    75    90   105   120

 .05    -
 .115   -***( 9 )
 .18    -
 .245   -*******( 21 )
 .31    -
 .375   -***********( 34 )
 .44    -
 .505   -*************( 40 )
 .57    -
 .635   -********************( 60 )
 .7     -
 .765   -****************************( 86 )
 .83    -
 .895   -*************************( 76 )
 .96    -
 1.025  -***********************( 71 )
 1.09   -
 1.155  -**************************************( 115 )
 1.22   -
 1.285  -************************************( 108 )
 1.35   -
 1.415  -****************************( 84 )
 1.48   -
 1.545  -***********************( 71 )
 1.61   -
 1.675  -************************( 73 )
 1.74   -
 1.805  -*********************( 63 )
 1.87   -
 1.935  -***********************( 69 )
 2.     -
 2.065  -( 0 )
 2.13   -
 2.195  -( 0 )
 2.26   -
 2.325  -( 0 )
 2.39   -

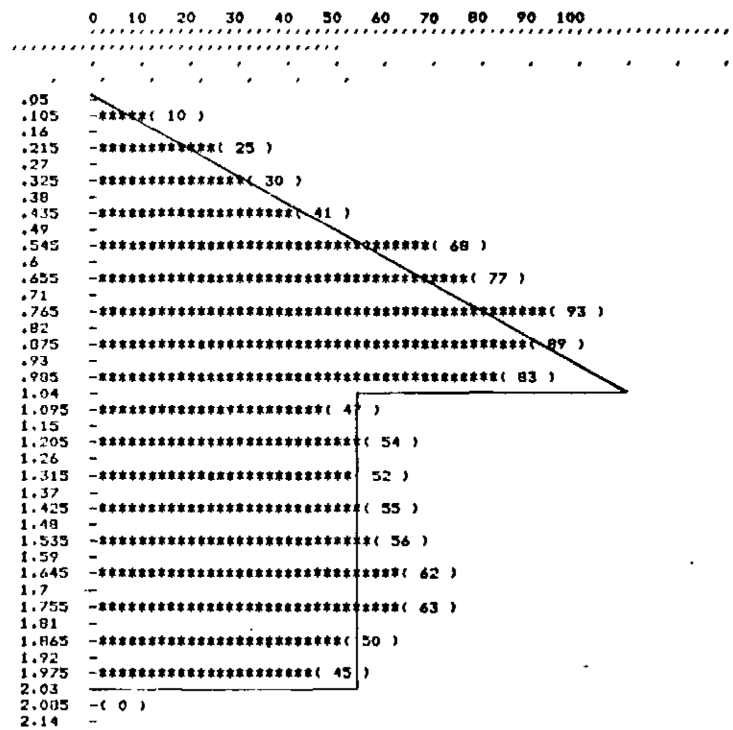
SRU       3.141 UNTS.

RUN COMPLETE.
```

SAMPLE MEAN= 1.13212
SAMPLE VARIANCE= .22441
STD. DEVIATION= .47372
MEDIAN= 1.09276
MODAL CLASS INTERVAL IS .95 TO 1.05
MODE= 1.00508
RANGE= 1.95974
MIDRANGE= 1.01941
MIDPOINT*FREQUENCY BASED MEAN= 1.132
MIDPOINT*FREQUENCY BASED VARIANCE= .225782
STD. DEV.= .475165



SAMPLE MEAN= 1.09587
SAMPLE VARIANCE= .246259
STD. DEVIATION= .496244
MEDIAN= 1.01946
MODAL CLASS INTERVAL IS .71 TO .82
MODE= .790
RANGE= 1.95974
MIDRANGE= 1.01941
MIDPOINT*FREQUENCY BASED MEAN= 1.09621
MIDPOINT*FREQUENCY BASED VARIANCE= .246662
STD. DEV.= .49665



SRU 3.119 UNTS.

```
SAMPLE MEAN= 1.06496
SAMPLE VARIANCE= .27162
STD. DEVIATION= .521172
MEDIAN= 1.01577
MODAL CLASS INTERVAL IS  .625  TO  .74
MODE= .681327
RANGE= 1.95974
MIDRANGE= 1.01941
MIDPOINT*FREQUENCY BASED MEAN= 1.06648
MIDPOINT*FREQUENCY BASED VARIANCE= .27165
STD. DEV.= .5212

        0   15   30   45   60   75   90  105  120

 .05    -
 .1075  -*****( 16 )
 .165   -
 .2225  -**********( 29 )
 .28    -
 .3375  -***************( 45 )
 .395   -
 .4525  -**********************( 65 )
 .51    -
 .5675  -****************************( 85 )
 .625   -
 .6825  -************************************( 109 )
 .74    -
 .7975  -****************************( 84 )
 .855   -
 .9125  -****************( 48 )
 .97    -
 1.0275 -****************( 49 )
 1.085  -
 1.1425 -*****************( 52 )
 1.2    -
 1.2575 -********************( 61 )
 1.315  -
 1.3725 -*****************( 53 )
 1.43   -
 1.4875 -*********************( 63 )
 1.545  -
 1.6025 -********************( 61 )
 1.66   -
 1.7175 -********************( 62 )
 1.775  -
 1.8325 -*******************( 58 )
 1.89   -
 1.9475 -********************( 60 )
 2.005
 2.0625 -( 0 )
 2.12   -

SRU      2.996 UNTS.

RUN COMPLETE.

 12.57.39. WARNING
```

```
SAMPLE MEAN= 1.04015
SAMPLE VARIANCE= .297674
STD. DEVIATION= .545597
MEDIAN= 1.00714
MODAL CLASS INTERVAL IS  .5  TO  .6
MODE= .536
RANGE= 1.95974
MIDRANGE= 1.01941
MIDPOINT*FREQUENCY BASED MEAN= 1.0407
MIDPOINT*FREQUENCY BASED VARIANCE= .29688
STD. DEV.= .544867

        0   10   20   30   40   50   60   70   80   90  100  110

 0      -
 .05    -***( 7 )
 .1     -
 .15    -************( 25 )
 .2     -
 .25    -******************( 37 )
 .3     -
 .35    -******************************( 60 )
 .4     -
 .45    -************************************( 73 )
 .5     -
 .55    -**************************************************( 100 )
 .6     -
 .65    -**************************( 52 )
 .7     -
 .75    -************************( 49 )
 .8     -
 .85    -*************************( 50 )
 .9     -
 .95    -**********************( 45 )
 1      -
 1.05   -*****************( 35 )
 1.1    -
 1.15   -************************( 49 )
 1.2    -
 1.25   -****************************( 56 )
 1.3    -
 1.35   -***********************( 46 )
 1.4    -
 1.45   -*************************( 51 )
 1.5    -
 1.55   -**************************( 52 )
 1.6    -
 1.65   -***************************( 55 )
 1.7    -
 1.75   -***************************( 55 )
 1.8    -
 1.85   -************************( 49 )
 1.9    -
 1.95   -***************************( 54 )
 2
 2.05   -( 0 )
 2.1    -
 2.15   -( 0 )
 2.2    -
```

```
SAMPLE MEAN= 1.02321
SAMPLE VARIANCE= .319728
STD. DEVIATION= .565445
MEDIAN= 1.00326
MODAL CLASS INTERVAL IS  .3  TO  .45
MODE= .370046
RANGE= 1.95774
MIDRANGE= 1.01941
MIDPOINT*FREQUENCY BASED MEAN= 1.02345
MIDPOINT*FREQUENCY BASED VARIANCE= .323754
STD. DEV.= .568994

        0   15   30   45   60   75   90  105  120

0
.075    -***********( 32 )
.15     -
.225    -*************************( 75 )
.3      -
.375    -***************************************( 116 )
.45     -
.525    -**************************( 79 )
.6      -
.675    -**************************( 79 )
.75     -
.825    -************************( 72 )
.9      -
.975    -***********************( 69 )
1.05    -
1.125   -********************( 60 )
1.2     -
1.275   -*************************( 76 )
1.35    -
1.425   -**************************( 77 )
1.5     -
1.575   -****************************( 83 )
1.65    -
1.725   -**************************( 79 )
1.8     -
1.875   -***********************( 70 )
1.95    -
2.025   -***********( 33 )
2.1
```

```
SAMPLE MEAN= 1.01199
SAMPLE VARIANCE= .337165
STD. DEVIATION= .580659
MEDIAN= 1.00714
MODAL CLASS INTERVAL IS  .1  TO  .2
MODE= .172308
RANGE= 1.97129
MIDRANGE= 1.01363
MIDPOINT*FREQUENCY BASED MEAN= 1.0118
MIDPOINT*FREQUENCY BASED VARIANCE= .338019
STD. DEV.= .581394

        0   10   20   30   40   50   60   70   80

0
.05     -**************( 27 )
.1      -
.15     -*************************************( 74 )
.2      -
.25     -****************************( 56 )
.3      -
.35     -**********************( 43 )
.4      -
.45     -**********************( 43 )
.5      -
.55     -******************************( 59 )
.6      -
.65     -**************************( 52 )
.7      -
.75     -*************************( 49 )
.8      -
.85     -*************************( 50 )
.9      -
.95     -***********************( 45 )
1       -
1.05    -******************( 35 )
1.1     -
1.15    -*************************( 49 )
1.2     -
1.25    -****************************( 56 )
1.3     -
1.35    -***********************( 46 )
1.4     -
1.45    -**************************( 51 )
1.5     -
1.55    -**************************( 52 )
1.6     -
1.65    -****************************( 55 )
1.7     -
1.75    -****************************( 55 )
1.8     -
1.85    -*************************( 49 )
1.9     -
1.95    -***************************( 54 )
2
2.05    -( 0 )
2.1     -
```

```
SAMPLE MEAN= 1.00843
SAMPLE VARIANCE= .34361
STD. DEVIATION= .586183
MEDIAN= 1.00595
MODAL CLASS INTERVAL IS  1.6  TO  1.8
MODE= 1.7
RANGE= 1.99039
MIDRANGE= 1.00008
MIDPOINT*FREQUENCY BASED MEAN= 1.0092
MIDPOINT*FREQUENCY BASED VARIANCE= .339215
STD. DEV.= .582421

        0    15   30   45   60   75   90  105  120
        ..........................................................
...........................................................

 0
 .1     -*************************************( 101 )
 .2     -
 .3     -************************************( 99 )
 .4     -
 .5     -*************************************( 102 )
 .6     -
 .7     -*************************************( 101 )
 .8     -
 .9     -**********************************( 95 )
 1      -
 1.1    -*******************************( 84 )
 1.2    -
 1.3    -*************************************( 102 )
 1.4    -
 1.5    -**************************************( 103 )
 1.6    -
 1.7    -****************************************( 110 )
 1.8    -
 1.9    -**************************************( 103 )
 2
```

Figure 4-9. The Statistical Distribution of Randomized Storage Dual Command Travel Time

SAMPLE MEAN= 1.79542
SAMPLE VARIANCE= .17656
STD. DEVIATION= .420191
MEDIAN= 1.8326
MODAL CLASS INTERVAL IS 1.88 TO 2
MODE= 1.91407
RANGE= 2.50061
MIDRANGE= 1.61126
MIDPOINT*FREQUENCY BASED MEAN= 1.79472
MIDPOINT*FREQUENCY BASED VARIANCE= .177005
STD. DEV.= .42072

0 15 30 45 60 75 90 105 120 135

.2	-
.26	-(0)
.32	-
.38	-*(3)
.44	-
.5	-*([illegible])
.56	-
.62	-*(3)
.68	-
.74	-*(5)
.8	-
.86	-***(10)
.92	-
.98	-****(13)
1.04	-
1.1	-*****(15)
1.16	-
1.22	-********(25)
1.28	-
1.34	-**************(42)
1.4	-
1.46	-********************(60)
1.52	-
1.58	-********************(62)
1.64	-
1.7	-************************(74)
1.76	-
1.82	-*********************************(100)
1.88	-
1.94	-***(123)
2	-
2.06	-*********************(65)
2.12	-
2.18	-*****************(51)
2.24	-
2.3	-*************(39)
2.36	-
2.42	-******(19)
2.48	-
2.54	-*****(16)
2.6	-
2.66	-****(13)
2.72	-
2.78	-**(7)
2.84	-
2.9	-(2)
2.96	-

SAMPLE MEAN= 1.69563
SAMPLE VARIANCE= .15987
STD. DEVIATION= .399837
MEDIAN= 1.73532
MODAL CLASS INTERVAL IS 1.71 TO 1.84
MODE= 1.77933
RANGE= 2.41914
MIDRANGE= 1.55696
MIDPOINT*FREQUENCY BASED MEAN= 1.69189
MIDPOINT*FREQUENCY BASED VARIANCE= .165277
STD. DEV.= .406543

0 10 20 30 40 50 60 70 80 90 100

.15	-
.215	-(0)
.28	-
.345	-*(2)
.41	-
.475	-*(3)
.54	-
.605	-*(3)
.67	-
.735	-***(6)
.8	-
.865	-***(7)
.93	-
.995	-*****(11)
1.06	-
1.125	-********(17)
1.19	-
1.255	-***********(23)
1.32	-
1.385	-*************************(51)
1.45	-
1.515	-***********************(46)
1.58	-
1.645	-*******************************(63)
1.71	-
1.775	-***(95)
1.84	-
1.905	-*********************************(67)
1.97	-
2.035	-********************(40)
2.1	-
2.165	-**************(28)
2.23	-
2.295	-******(12)
2.36	-
2.425	-******(12)
2.49	-
2.555	-****(9)
2.62	-
2.685	-**(4)
2.75	-

SRU 2.036 UNTS.

RUN COMPLETE.
REWIND,FH09

```
SAMPLE MEAN= 1.63919
SAMPLE VARIANCE= .158459
STD. DEVIATION= .398069
MEDIAN= 1.65791
MODAL CLASS INTERVAL IS  1.58  TO  1.71
MODE= 1.658
RANGE= 2.4498
MIDRANGE= 1.44259
MIDPOINT*FREQUENCY BASED MEAN= 1.6385
MIDPOINT*FREQUENCY BASED VARIANCE= .158063
STD. DEV.= .397572

         0   15   30   45   60   75   90  105  120  135  150

 .15     -
 .215    -( 1 )
 .28     -
 .345    -( 2 )
 .41     -
 .475    -**( 6 )
 .54     -
 .605    -*( 4 )
 .67     -
 .735    -****( 12 )
 .8      -
 .865    -******( 20 )
 .93     -
 .995    -**********( 31 )
 1.06    -
 1.125   -******************( 54 )
 1.19    -
 1.255   -************************( 73 )
 1.32    -
 1.385   -*****************************( 88 )
 1.45    -
 1.515   -****************************************( 122 )
 1.58    -
 1.645   -************************************************( 146 )
 1.71    -
 1.775   -*******************************************( 130 )
 1.84    -
 1.905   -****************************************( 121 )
 1.97    -
 2.035   -****************************( 84 )
 2.1     -
 2.165   -***************( 45 )
 2.23    -
 2.275   -*********( 29 )
 2.36    -
 2.425   -*****( 16 )
 2.49    -
 2.555   -****( 14 )
 2.62    -
 2.685   -( 2 )
 2.75    -
```

```
SAMPLE MEAN= 1.5699
SAMPLE VARIANCE= .160952
STD. DEVIATION= .401188
MEDIAN= 1.6042
MODAL CLASS INTERVAL IS  1.59  TO  1.73
MODE= 1.67244
RANGE= 2.3553
MIDRANGE= 1.39081
MIDPOINT*FREQUENCY BASED MEAN= 1.57138
MIDPOINT*FREQUENCY BASED VARIANCE= .162064
STD. DEV.= .402572

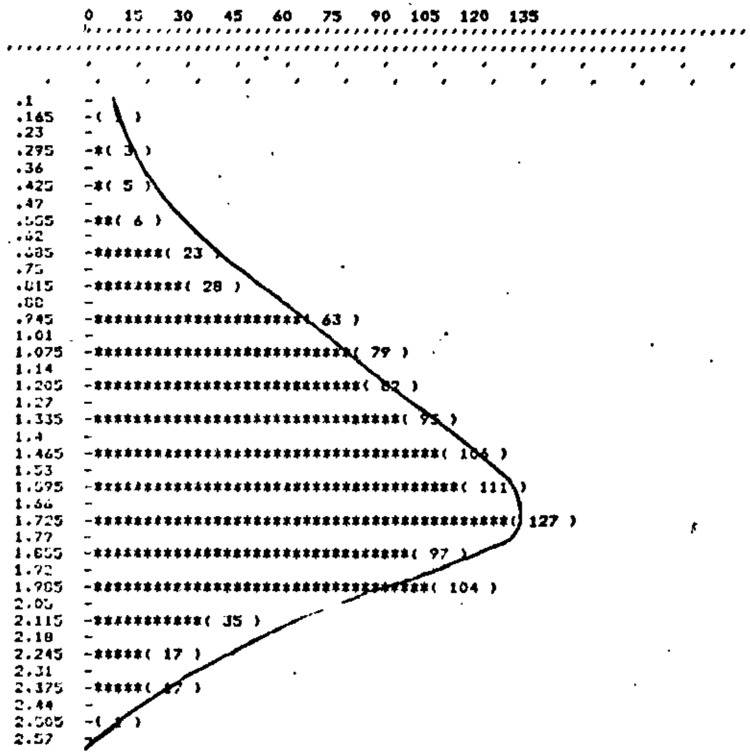
         0   15   30   45   60   75   90  105  120  135  150

 .05     -
 .12     -( 0 )
 .19     -
 .26     -*( 3 )
 .33     -
 .4      -*( 4 )
 .47     -
 .54     -*( 4 )
 .61     -
 .68     -*****( 15 )
 .75     -
 .82     -********( 25 )
 .89     -
 .96     -*************( 40 )
 1.03    -
 1.1     -***************************( 82 )
 1.17    -
 1.24    -************************( 74 )
 1.31    -
 1.38    -******************************************( 126 )
 1.45    -
 1.52    -*************************************( 113 )
 1.59    -
 1.66    -***********************************************( 143 )
 1.73    -
 1.8     -********************************************( 132 )
 1.87    -
 1.94    -*****************************************( 123 )
 2.01    -
 2.08    -*****************( 53 )
 2.15    -
 2.22    -***********( 34 )
 2.29    -
 2.36    -******( 18 )
 2.43    -
 2.5     -****( 14 )
 2.57    -

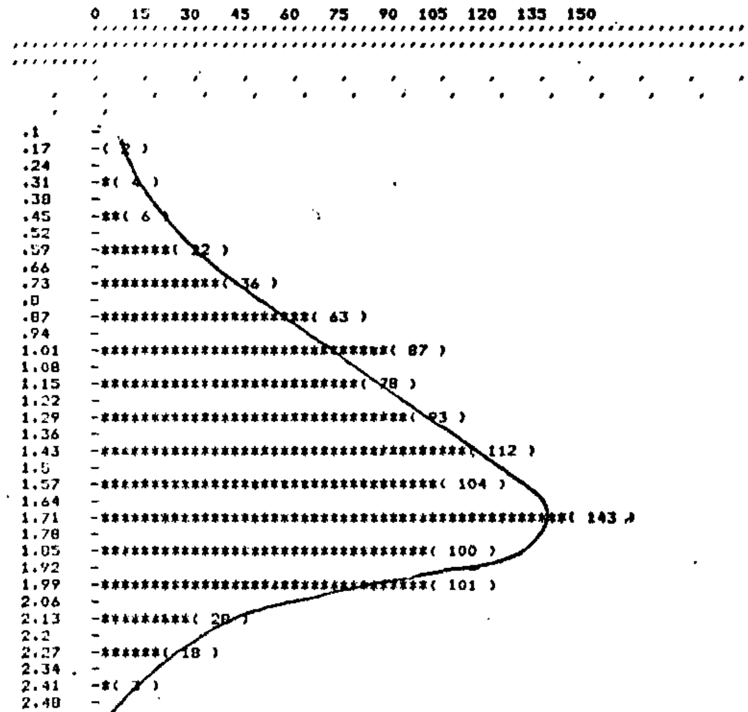
SRU       3.147 UNTS.

RUN COMPLETE.
```

SAMPLE MEAN= 1.50936
SAMPLE VARIANCE= .168556
STD. DEVIATION= .410556
MEDIAN= 1.54113
MODAL CLASS INTERVAL IS 1.66 TO 1.79
MODE= 1.70522
RANGE= 2.25633
MIDRANGE= 1.34125
MIDPOINT*FREQUENCY BASED MEAN= 1.50803
MIDPOINT*FREQUENCY BASED VARIANCE= .170141
STD. DEV.= .412401



SAMPLE MEAN= 1.45818
SAMPLE VARIANCE= .179247
STD. DEVIATION= .423376
MEDIAN= 1.49687
MODAL CLASS INTERVAL IS 1.64 TO 1.78
MODE= 1.70657
RANGE= 2.15727
MIDRANGE= 1.29173
MIDPOINT*FREQUENCY BASED MEAN= 1.45758
MIDPOINT*FREQUENCY BASED VARIANCE= .181446
STD. DEV.= .425965



SRU 3.086 UNTS.

```
SAMPLE MEAN= 1.41548
SAMPLE VARIANCE= .191048
STD. DEVIATION= .43709
MEDIAN= 1.47075
MODAL CLASS INTERVAL IS  1.65  TO  1.75
MODE= 1.67143
RANGE= 2.05953
MIDRANGE= 1.24158
MIDPOINT*FREQUENCY BASED MEAN= 1.4102
MIDPOINT*FREQUENCY BASED VARIANCE= .196372
STD. DEV.= .443162

        0    10   20   30   40   50   60   70   80   90  100

 .05    -
 .1     -( 0 )
 .15    -
 .2     -*( 3 )
 .25    -
 .3     -**( 4 )
 .35    -
 .4     -**( 4 )
 .45    -
 .5     -******( 13 )
 .55    -
 .6     -*****************( 31 )
 .65    -
 .7     -********************( 36 )
 .75    -
 .8     -**********************( 40 )
 .85    -
 .9     -****************************( 51 )
 .95    -
 1.     -*****************************( 53 )
 1.05   -
 1.1    -******************************( 54 )
 1.15   -
 1.2    -************************( 44 )
 1.25   -
 1.3    -***************************************( 71 )
 1.35   -
 1.4    -*********************************************( 81 )
 1.45   -
 1.5    -*****************************************( 74 )
 1.55   -
 1.6    -*************************************************( 88 )
 1.65   -
 1.7    -****************************************************( 94 )
 1.75   -
 1.8    -****************************************( 72 )
 1.85   -
 1.9    -**********************************************( 83 )
 1.95   -
 2.     -**************************************( 69 )
 2.05   -
 2.1    -**********( 18 )
 2.15   -
 2.2    -********( 15 )
 2.25   -
```

```
SAMPLE MEAN= 1.38207
SAMPLE VARIANCE= .203214
STD. DEVIATION= .450793
MEDIAN= 1.45231
MODAL CLASS INTERVAL IS  1.675  TO  1.8
MODE= 1.7375
RANGE= 2.03221
MIDRANGE= 1.17496
MIDPOINT*FREQUENCY BASED MEAN= 1.38275
MIDPOINT*FREQUENCY BASED VARIANCE= .205233
STD. DEV.= .453026

        0   15   30   45   60   75   90  105  120

 .05    -
 .1125  -( 1 )
 .175   -
 .2375  -*( 4 )
 .3     -
 .3625  -***( 11 )
 .425   -
 .4875  -*************( 34 )
 .55    -
 .6125  -*****************( 45 )
 .675   -
 .7375  -**************( 37 )
 .8     -
 .8625  -********************( 52 )
 .925   -
 .9875  -**************************( 69 )
 1.05   -
 1.1125 -************************( 63 )
 1.175  -
 1.2375 -**************************( 68 )
 1.3    -
 1.3625 -************************************( 94 )
 1.425  -
 1.4875 -****************************************( 103 )
 1.55   -
 1.6125 -**************************************( 98 )
 1.675  -
 1.7375 -********************************************( 114 )
 1.8    -
 1.8625 -**************************************( 98 )
 1.925  -
 1.9875 -***********************************( 90 )
 2.05   -
 2.1125 -******( 18 )
 2.175  -
 2.2375 -( 1 )
 2.3    -

SRU      3.272 UNTS.

RUN COMPLETE.
```

SAMPLE MEAN= 1.35918
SAMPLE VARIANCE= .212172
STD. DEVIATION= .460622
MEDIAN= 1.43603
MODAL CLASS INTERVAL IS 1.69 TO 1.82
MODE= 1.74474
RANGE= 1.99762
MIDRANGE= 1.11671
MIDPOINT*FREQUENCY BASED MEAN= 1.36175
MIDPOINT*FREQUENCY BASED VARIANCE= .214581
STD. DEV.= .463220

0 15 30 45 60 75 90 105 120

0	-
.065	-(1)
.13	-
.195	-*(4)
.26	-
.325	-********(24)
.39	-
.455	-*********(27)
.52	-
.585	-*************(40)
.65	-
.715	-***************(44)
.78	-
.845	-******************(54)
.91	-
.975	-********************(59)
1.04	-
1.105	-**********************(64)
1.17	-
1.235	-**************************(78)
1.3	-
1.365	-**********************************(101)
1.43	-
1.495	-*********************************(97)
1.56	-
1.625	-*************************************(110)
1.69	-
1.755	-**(118)
1.82	-
1.885	-************************************(107)
1.95	-
2.015	-************************(70)
2.08	-
2.145	-(2)
2.21	-

SAMPLE MEAN= 1.34536
SAMPLE VARIANCE= .218542
STD. DEVIATION= .467485
MEDIAN= 1.42675
MODAL CLASS INTERVAL IS 1.69 TO 1.82
MODE= 1.77667
RANGE= 1.93015
MIDRANGE= 1.07407
MIDPOINT*FREQUENCY BASED MEAN= 1.34797
MIDPOINT*FREQUENCY BASED VARIANCE= .220966
STD. DEV.= .47007

0 15 30 45 60 75 90 105 120

0	-
.065	-(1)
.13	-
.195	-****(12)
.26	-
.325	-********(22)
.39	-
.455	-**********(29)
.52	-
.585	-*************(39)
.65	-
.715	-*****************(49)
.78	-
.845	-****************(46)
.91	-
.975	-*********************(61)
1.04	-
1.105	-***********************(68)
1.17	-
1.235	-*************************(76)
1.3	-
1.365	-**********************************(100)
1.43	-
1.495	-********************************(95)
1.56	-
1.625	-*************************************(111)
1.69	-
1.755	-**(117)
1.82	-
1.885	-***************************************(114)
1.95	-
2.015	-********************(60)
2.08	-

```
SAMPLE MEAN= 1.34092
SAMPLE VARIANCE= .220644
STD. DEVIATION= .469727
MEDIAN= 1.425
MODAL CLASS INTERVAL IS  1.9  TO  2
MODE= 1.9203
RANGE= 1.71606
MIDRANGE= 1.04147
MIDPOINT*FREQUENCY BASED MEAN= 1.3406
MIDPOINT*FREQUENCY BASED VARIANCE= .220272
STD. DEV.= .469331

        0    15    30    45    60    75    90   105   120

0       -
.05     -( 1 )
.1      -
.15     -*( 4 )
.2      -
.25     -******( 19 )
.3      -
.35     -*****( 15 )
.4      -
.45     -*******( 23 )
.5      -
.55     -***********( 34 )
.6      -
.65     -*******( 22 )
.7      -
.75     -**************( 42 )
.8      -
.85     -************( 36 )
.9      -
.95     -***************( 47 )
1       -
1.05    -*****************( 53 )
1.1     -
1.15    -****************( 49 )
1.2     -
1.25    -********************( 61 )
1.3     -
1.35    -*************************( 76 )
1.4     -
1.45    -************************( 74 )
1.5     -
1.55    -*************************( 77 )
1.6     -
1.65    -******************************( 90 )
1.7     -
1.75    -******************************( 92 )
1.8     -
1.85    -**************************( 79 )
1.9     -
1.95    -***********************************( 106 )
2
```

for dual command. The particular distribution under question will depend upon the number of product classes and the spread of their C/A ratios.

Summary

The S/R travel time is an important variable in terms of meeting the throughput required from the system. This chapter has analyzed mean travel time and its variance both for randomized and dedicated storage. Closed form expressions were obtained for randomized storage travel time. Simulation has been used to approximate dedicated storage travel time. In addition, histograms were presented for travel time under randomized storage.

CHAPTER V

THE OPTIMIZATION MODEL

Introduction

This chapter will present the mathematical model of the AS/R system. Following a discussion of the model, the cost elements introduced in Chapter II, will be analyzed in terms of convexity. In addition, the expected throughput level is analyzed as a function of the shape factor. Based on the above findings, an optimization algorithm will be developed and presented in full detail. Recall the notation previously developed, where:

λ	= throughput level demanded from the system
TNOA	= total number of openings
NOAI	= number of storage aisles
NOL	= number of levels in each aisle
ICOL	= number of columns in each aisle
DEPTH	= unit load depth
WTH	= unit load width
HEHT	= unit load height
hv	= horizontal travel velocity of the S/R
vv	= vertical travel velocity of the S/R
BHEHT	= building height
BLTH	= building length
BWTH	= building width

IADD = amount of clearance provided at the end of the aisles. It is added to rack length in order to determine the building length

BCOST = unit cost (in dollars/sq.ft.) to construct a 25' high free standing building

CFR = the conversion factor used to compute the unit construction cost of a building with a height greater than 25'

LCOST = unit land cost (in dollars/sq.ft.)

The Mathematical Model

The system under question can be modeled as follows:

min TC = Land Cost + Building Cost + Rack Cost + S/R Cost

+ Total Annual Recurring Costs (P/A, i%, 10)

s.t.

(expected throughput level) $\geq \lambda$

$2 \times NOAI \times NOL \times ICOL \geq TNOA$

The possibility of solving the above model with classical optimization techniques can be discussed with respect to the storage method used. First, consider dedicated storage. It was not possible to develop closed form expressions for the expected travel time (Chapter IV). Hence, it is not possible to state the first constraint in a closed form. The closed form expressions for travel time with randomized storage were presented in Chapter IV. In terms of the model variables the shape factor, b, will be:

$$b = \min\left\{\frac{(\text{ICOL x WTH})/hv}{(\text{NOL x HEHT})/vv}, \frac{(\text{NOL x HEHT})/vv}{(\text{ICOL x WTH})/hv}\right\}$$

Recall that the expressions were obtained for a normalized rack (longer travel time being unity). Hence, an inverse transformation was required to find the true expected travel time for the rack under question. As a result, the throughput constraint has a complex form. Furthermore, when waiting time is included in determining the throughput, the variance of travel time also has to be considered and the expression for expected throughput will get further complicated. In addition, note that both the objective function and the constraints are non-linear in NOL, ICOL and NOAI. Consequently, solving the entire model with classical optimization techniques does not seem to be promising. However, we can initially fix one of the variables (NOAI) and ignore the throughput constraint in order to see how the cost functions behave in terms of the remaining variables, namely NOL and ICOL.

Analysis of Cost Elements

Assuming that the nnmber of aisles is fixed, individual cost elements can be analyzed in terms of convexity. The following analysis assumes that the variables NOL and ICOL are continuous.

The Land Cost: Recall the expression for land cost as

$$c_L = \text{BLTH x BWTH x LCOST}$$

$$c_L = f(\text{ICOL}) \text{ x } g(\text{NOAI}) \text{ x LCOST}$$

let K = g(NOAI) x LCOST, then

$$c_L = f(ICOL) \times K = \left[(ICOL \times WTH) + IADD\right] \times K$$

let G = number of openings per rack = TNOA/2 x NOAI (a fixed number)

then

$$NOL \times ICOL = G$$

$$\therefore \quad c_L = \left[\frac{G}{NOL} \times WTH \times K\right] + IADD \times K$$

$$\frac{dc_L}{d(NOL)} = -\frac{1}{NOL^2}\left[G \times WTH \times K\right] \tag{5-1}$$

Since NOL > 0 and (G x WTH x K) > 0, $dC_L/d(NOL) < 0$. Furthermore,

$$\frac{d^2c_L}{D(NOL)^2} = 2(NOL)^{-3}\left[G \times WTH \times K\right] > 0$$

Therefore, land cost is a convex decreasing function of NOL.

The Building Cost: Recall the expression for building cost as:

$$c_B = BLTH \times BWTH \times BCOST \times CFR$$

also, recall the expression for CFR as:

$$CFR = 0.986508 - 0.005349BHEHT + 0.0002698(BHEHT)^2$$

$$\text{let} \quad CFR = c_1 - (c_2 \times BHEHT) + c_3(BHEHT)^2$$

$$\therefore \quad c_B = \left[(ICOL \times WTH) + IADD\right] \times BWTH \times BCOST \times \left[c_1 - (c_2 \times BHEHT) + c_2(BHEHT)^2\right]$$

Substitute ICOL = G/NOL and BHEHT = (NOL x HEHT) + 4.0 in the above expression for c_B. Upon opening the parentheses one obtains:

$$c_B = c_1 \cdot \frac{G}{NOL} \text{ WTH . BWTH . BCOST} + c_1 \text{ . IADD . BWTH . BCOST}$$

$$- c_2 \cdot \frac{G}{NOL} \text{ . WTH . BWTH . BCOST} \left[(\text{NOL . HEHT}) + 4\right]$$

$$- c_2 \text{ . IADD . BWTH . BCOST} \left[(\text{NOL . HEHT}) + 4\right]$$

$$+ c_3 \cdot \frac{G}{NOL} \text{ . WTH . BWTH . BCOST}(\text{NOL}^2 \text{ . HEHT}^2 + 8 \text{ . NOL . HEHT} + 16)$$

$$+ c_3 \text{ . IADD . BWTH . BCOST}(\text{NOL}^2 \text{ . HEHT}^2 + 8\text{NOL . HEHT} + 16)$$

Recall that IADD, BWTH and BCOST are constant. Hence, if we add the constant terms and let

$$\gamma_1 = \text{G . WTH . BWTH . BCOST}$$

$$\gamma_2 = \text{HEHT . IADD . BWTH . BCOST}$$

we have

$$\begin{aligned} c_B = {} & (\text{NOL})^{-1}(c_1\gamma_1 - 4c_2\gamma_1 + 16c_3\gamma_1) \\ & + \text{NOL}(c_3\gamma_1(\text{HEHT})^2 - c_2\gamma_2 + 8c_3\gamma_2) \\ & + \text{NOL}^2 \, (c_3 \text{ . HEHT . } \gamma_2) + \text{constants} \end{aligned} \tag{5-2}$$

Hence,

$$\frac{dc_B}{d(\text{NOL})} = -(\text{NOL})^{-2}(c_1\gamma_1 - 4c_2\gamma_1 + 16c_3\gamma_1) + c_3\gamma_1(\text{HEHT})^2 - c_2\gamma_2 + 8c_3\gamma_2 + 2(\text{NOL})(c_3 \,.\, \text{HEHT} \,.\, \gamma_2)$$

Hence, $d^2c_B/d(\text{NOL})^2 = 2(\text{NOL})^{-3}(c_1\gamma_1 - 4c_2\gamma_1 + 16\ c_3\gamma_1) + 2(c_3 \,.\, \text{HEHT} \,.\, \gamma_2)$

The building cost will be convex if $d^2c_B/d(\text{NOL})^2 > 0$; that is if the following hold:

$$(\text{NOL})^{-3}(c_1\gamma_1 - 4c_2\gamma_1 + 16c_3\gamma_1) > 0$$

and $$(c_3 \,.\, \text{HEHT} \,.\, \gamma_2) > 0$$

Consider the first inequality. We know that NOL > 0. Therefore we have to verify that $c_1\gamma_1 - 4c_2\gamma_1 + 16c_3\gamma_1 > 0$ holds; i.e. is $c_1\gamma_1 + 16c_2\gamma_1 > 4c_2\gamma_1$.
but

$$\gamma_1 = G \,.\, \text{WTH} \,.\, \text{BWTH} \,.\, \text{BCOST} > 0$$

i.e. is $$c_1 + 16c_3 > 4c_2$$

is $$0.986508 + 16(0.0002698) > 4(0.005349)$$

$$0.990324 \quad > \quad 0.021396$$

Thus $(NOL)^{-3}\ (c_1\gamma_1 - 4c_2\gamma_1 + 16c_3\gamma_1)$ is strictly positive. Now consider the second inequality. Recall

$$\gamma_2 = HEHT\ .\ IADD\ .\ BWTH\ .\ BCOST > 0$$

$$\text{Also} \quad c_3 = 0.0002698 \text{ and } HEHT > 0$$

Therefore, $$d^2c_B/d(NOL)^2 > 0$$

Hence, building cost is a convex function of NOL. [Note that, if it was not possible to show that the first inequality was true, one would attempt to show that $(c_3\ .\ HEHT\ .\ \gamma_2) > (NOL)^{-3}\ .\ (c_1\gamma_1 - 4c_2\gamma_1 + 16c_3\gamma_1)$, where NOL will be assumed to take its maximum value].

<u>The Rack Cost</u>: Recall from Chapter II that, total rack cost, c_R, is

$$\begin{aligned} c_R &= RCOST \times TNOA \\ &= \left[RCOST + (0.23328NOL) - (0.00476 \times NOL^2)\right] \times 14.0 \times TNOA \end{aligned}$$

In the above expression RCOST is fixed because it is a function of ALS (unit load size) and WEGT (unit load weight). Also, TNOA = 2 x NOAI x ICOL x NOL. Therefore, $c_R = 28 \times NOL \times \frac{G}{NOL} \times NOAI \left[RCOST + 0.23328NOL - 0.00476NOL^2\right]$.

Since NOAI is also fixed, the variable rack cost, c_{VR}, will be

$$c_{VR} = 28(0.23328)G\ .\ NOAI(NOL) - 28(0.00476)G\ .\ NOAI(NOL)^2$$

The above function is concave in NOL, due to the second term. Recall the original equation (from Chapter II):

$$\text{variable cost/rack opening} = 0.23328z - 0.00476z^2$$

where z is the height of the rack expressed in unit loads. The intercept of the above line equals to 0.142857, which is included in 0.92484 that appears in the cost/rack opening equation in Chapter II. An alternative approach would be to drop the z^2 term and re-estimate the coefficients. The least-squares estimate from the data in Table 2-3 gives the equation:

$$\text{variable cost/rack opening} = 0.797468 + 0.0949367z$$

Adjusting the original line for the above modified variable cost, we obtain

$$\text{cost/rack opening} = \$14\left[1.579451 + 0.025x + 0.0004424y - \frac{y^2}{82{,}500{,}000} + 0.0949367z\right]$$

For the example presented in Chapter 2, the above modified line gives \$65.73/opening instead of \$70/opening obtained from Table 2-3. Hence, in the Fibonacci search, the above convex expression for rack cost was used while the original expression was kept in enumerating over the final interval of uncertainty. Note that the variable cost/rack opening is now an increasing convex function of z, where z corresponds to NOL. Furthermore, the variable rack cost, c_{VR}, will be:

$$c_{VR} = 28(0.0949367) \times NOL \times G \times NOAI = 2.6582 \times NOL \times G \times NOAI \quad (5\text{-}3)$$

The S/R Cost: Recall that the S/R cost is a function of the height of the AS/RS, the weight of the unit load, and the type and location of the control logic. In terms of the design variables, the variable S/R cost is a linear piecewise function of the rack height where the break-points are the number of levels corresponding to 30', 42', 60' and 85'.

The Recurring Costs: The annual maintenance cost on the building was stated as:

$$c_{BM} = BLTH \times BWTH \times BMCOST$$

Note that the above expression is very similar to land cost, with the exception that it is a recurring cost. Hence, for c_{BM} we have

$$c_{BM} = \left\{ \left[\frac{G}{NOL} \times WTH \times M \right] + \left[IADD \times M \right] \right\} (P/A, i\%, n) \quad (5\text{-}4)$$

where $G = NOL \times ICOL$

$M = g(NOAI) \times BMCOST$

It was previously shown that land cost is a convex decreasing function of NOL. Therefore, so is the building maintenance cost.

The Total Cost: The land cost and the building maintenance cost are convex decreasing functions of NOL. The rack cost is a convex increasing function of NOL. Hence, by adding the above cost elements one obtains a convex function in NOL. say h_1(NOL). The building cost is also a convex function of NOL. Hence, adding building cost to h_1(NOL) will produce a convex function in NOL, say h_2(NOL). Recall that S/R cost is a linear piecewise function of rack height with four break-points, say b_1, b_2, b_3 and b_4 (see Figure 5-1). When the S/R cost function is added to h_2(NOL), the resulting function, TC(NOL), i.e. the total system cost will be a piece-wise convex function of NOL. In terms of the minimum point, the total cost function may take different forms depending on the magnitude of the step size at the break-points and the slope of $h_2(x)$, (see Figures 5-1a and 5-1b). Note that, in a general case there may exist different step sizes at different break-points, and therefore one can develop several different forms of the total cost function. But, in any case the following rule will be true: if we let h' denote the number of levels obtained by minimizing h_2(NOL), then the number of levels that minimizes the total cost function, say NOL_1, will either correspond to h' or it will be equal to one of the break-points that lie to the "left" of h'. In terms of Figures (5-1a) and (5-1b) the above rule states that NOL_1 equals either to h', b_1 or b_2. Of course, if $h' > b_4$, then all four break-points are candidates for determining NOL_1.

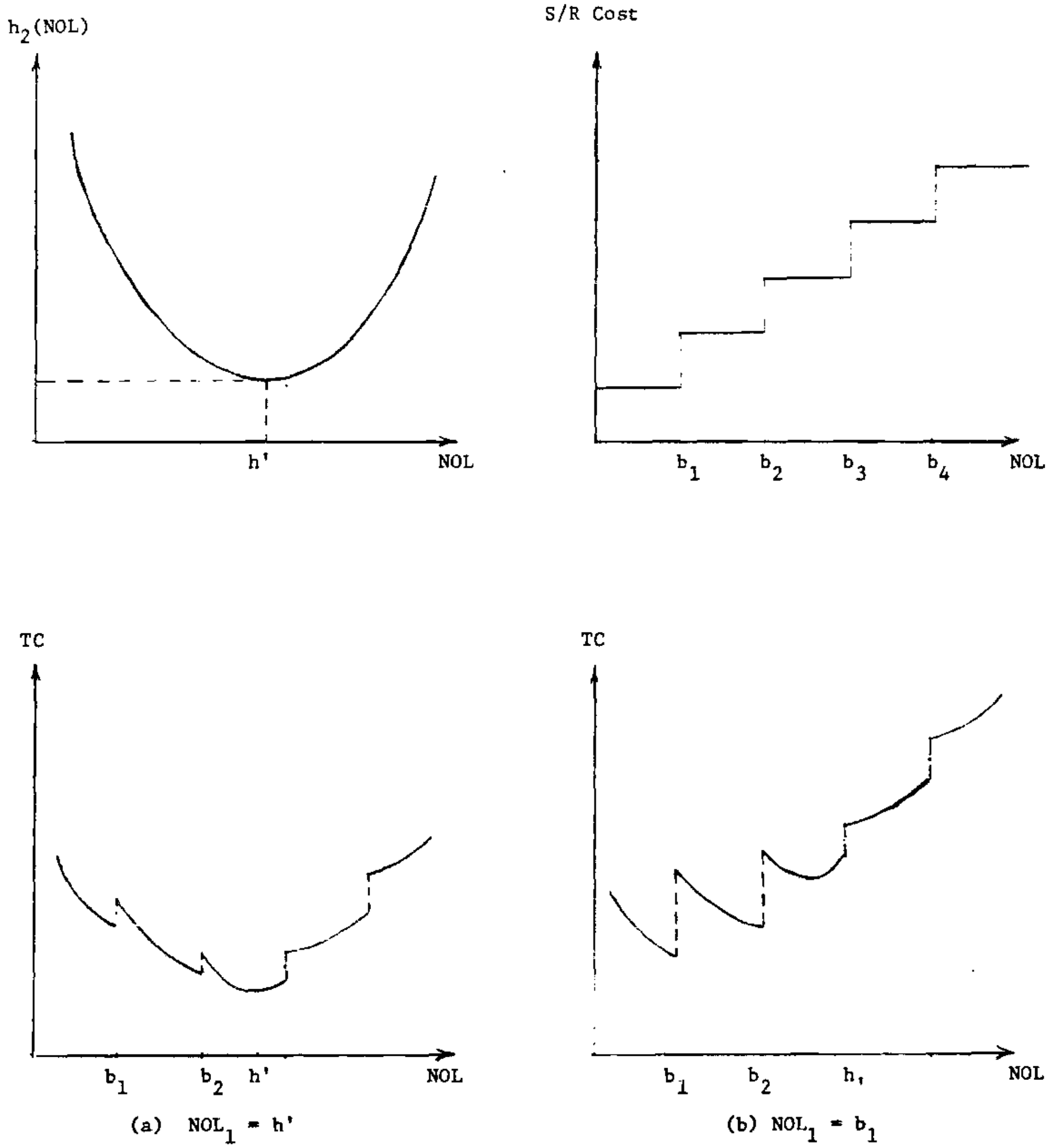


Figure 5-1. The Total Cost Function
for Fixed Number of Aisles

For a given unit load width (WTH), the b_i's are determined as follows, let:

$$WTHP = (WTH + 8")/12.0 \text{ ft.}$$

and $[a]$ = the greatest integer less than or equal to a.

Then

$$b_i = \left[(y_i/WTHP) + 0.5\right]$$

where y_i = 30, 32, 60 and 85 for i=1, 2, 3 and 4, respectively. Determination of h' is as follows. Recall that h' minimizes $h_2(NOL)$ where $h_2(NOL)$ is the total land, building, rack and maintenance cost. The land cost is given by (5-1) as:

$$c_L = \left[\frac{G}{NOL} \times WTH \times K\right] + \left[IADD \times K\right]$$

where $$G = NOL \times ICOL$$

and $$K = g(NOAI) \times LCOST = BWTH \times LCOST$$

Let $$\gamma_3 = G \times WTH \times BWTH \times LCOST$$

then $$c_L = \frac{\gamma_3}{NOL} + \left[IADD \times K\right] \qquad (5\text{-}5)$$

The maintenance cost on the building is given by (5-4). Let

$$\gamma_4 = G \times WTH \times BWTH \times BMCOST \times (P/A,10\%,10)$$

then $$c_{BM} = \frac{\gamma_4}{NOL} + \left[IAD \times M \times (P/A,10\%,10)\right] \qquad (5\text{-}6)$$

Next consider the building cost given by (5-2) and the variable rack cost given by (5-3). If we delete the constant terms (IADD x K) in (5-5) and (IADD x M x (P/A,10%,10)) in (5-6) and add (5-5), (5-2), (5-3) and (5-6), after rearranging the terms, we obtain $h_2(\mathrm{NOL})$ as follows:

$$h_2(\mathrm{NOL}) = \frac{1}{\mathrm{NOL}}(c_1\gamma_1 - 4c_2\gamma_1 + 16c_3\gamma_1 + \gamma_3 + \gamma_4)$$
$$+ \mathrm{NOL}(c_3\gamma_1(\mathrm{HEHT})^2 - c_2\gamma_2 + 8c_3\gamma_2 + 2.6582 \times G \times \mathrm{NOAI})$$
$$+ \mathrm{NOL}^2(c_3 \cdot \mathrm{HEHT} \cdot \gamma_2) \qquad (5\text{-}7)$$

In the above equation, let

$$\alpha_1 = c_1\gamma_1 - 4c_2\gamma_1 + 16c_3\gamma_1 + \gamma_3 + \gamma_4$$

$$\alpha_2 = c_3\gamma_1(\mathrm{HEHT})^2 - c_2\gamma_2 + 8c_3\gamma_2 + 2.6582 \times G \times \mathrm{NOAI}$$

and $$0.5\alpha_3 = c_3(\mathrm{HEHT})\gamma_2$$

Taking the first derivative with respect NOL, one obtains:

$$\frac{dh_2(\mathrm{NOL})}{d(\mathrm{NOL})} = -\frac{1}{\mathrm{NOL}^2}\ \alpha_1 + \alpha_2 + (\mathrm{NOL})\alpha_3$$

Recall that $h_2(\mathrm{NOL})$ should be minimized at h'. Therefore, we have

$$-\frac{1}{(h')^2}\ \alpha_1 + \alpha_2 + (h')\alpha_3 = 0$$

Hence, h' satisfies

$$(h')^2\alpha_2 + (h')^3\alpha_3 = \alpha_1$$

Since convexity of $h_2(NOL)$ has previously been established, we do not need to check $h_2''(NOL)$. Furthermore, note that the S/R maintenance and labor cost are not included in determining NOL_1 because they are not a function of the number of levels, once the number of aisles is fixed.

It has been demonstrated that, given a fixed number of aisles, the number of levels that minimize total system cost, namely NOL_1, will either be equal to h' obtained from (5-8) or one of the b_i's to the left of h'. Mathematically,

$$NOL_1 = \min_{b_i < h'} \left\{ TC(h') \;,\; TC(b_i) \right\} \tag{5-9}$$

Note that the throughput constraint has not been considered in the above approach. In other words, we are not guaranteed that the required throughput will be met with the rack obtained by setting $NOL = NOL_1$. The analysis of throughput as a function of the shape factor is presented in the following section.

Analysis of Throughput

In analyzing the effect of throughput requirements on the optimization, it is instructive to first consider the randomized storage case. It will be shown that the expected single command travel time is minimized when b=1. Suppose we have a rack for which b=1, i.e. H/vv = L/hv and LH = K (where K - required rack area). Hence,

$$E(\overline{SC}/b = 1) = (\frac{b^2}{3} + 1)\ \frac{L}{hv} = 4L/3hv \qquad (5\text{-}10)$$

Now, suppose we increase the length of the rack to $L' = cL$, where $c \gtrsim 1$; the new height H' will be:

$$H' = \frac{K}{L'} = \frac{K}{CL} = \frac{KH}{cK} = \frac{H}{c}$$

$$\therefore \quad b = \frac{H'/vv}{L'/hv} = \frac{H\ .\ hv}{c^2\ .\ L\ .\ vv} \quad \text{but } H\ .\ hv = L\ .\ vv$$

$$\therefore \quad b = \frac{1}{c^2}$$

$$\therefore \quad E(SC/b{=}c^{-2}) = (\frac{1}{3c^4} + 1)\ \frac{cL}{hv} \qquad (5\text{-}11)$$

Now, compare (5-10) and (5-11). We need to show

$$(\frac{1}{3c^4} + 1)\frac{cL}{hv} \geq \frac{4L}{3hv} \quad \text{for all } c \geq 1$$

$$\text{i.e.} \qquad \frac{1}{3c^3} + 6 \geq 1 + \frac{1}{3} \quad \text{for all } c \geq 1$$

The above equation becomes an equality at c=1, and for any c>1, the left-hand side will be greater than $(1 + \frac{1}{3})$. The proof is very similar if we increase the rack height instead of the rack length. In any case, the single command expected travel time is minimized at b=1.

The variance of single command travel time, $V(\overline{SC})$, will also be minimized at b=1. Recall that

$$V(\overline{SC}) = \frac{2}{3} b^3 + \frac{4}{3} - E^2(\overline{SC})$$

$$\therefore \quad V(\overline{SC}/b=1) = \frac{34}{9} \left(\frac{L}{hv}\right)^2 \tag{5-12}$$

Also,

$$V(\overline{SC}/b=c^{-2}) = \left(\frac{1}{9c^8} + \frac{2}{3c^6} + \frac{2}{3c^4} + \frac{7}{3}\right) c^2 \left(\frac{L}{hv}\right)^2 \tag{5-13}$$

In terms of (5-12) and (5-13) we need to show that

$$\frac{1}{9c^6} + \frac{2}{3c^4} + \frac{2}{3c^2} + \frac{7}{3} c^2 \geq \frac{34}{9}$$

At c=1 the left-hand side of above inequality becomes 34/9, for any c>1 it is always greater than the right-hand side.

Next consider the expected value of dual command travel time, namely $E(\overline{DC})$. From (4-17) we have

$$E'(\overline{DC}) = \frac{4}{3} + \frac{1}{2} b^2 - \frac{1}{30} b^3$$

$$\therefore \quad E(\overline{DC})/b=1) = 1.80 \frac{L}{hv} \tag{5-14}$$

Also $$E(\overline{DC}/b=c^{-2}) - (\frac{4}{3} + \frac{1}{2c^4} - \frac{1}{30c^6})\frac{cL}{hv} \qquad (5\text{-}15)$$

In terms of (5-14) and (5-15) we need to show

$$(\frac{4}{3}c + \frac{1}{2c^3} - \frac{1}{30c^5}) \geq 1.80$$

The left-hand side of the above inequality equals 1.80 when c=1; for c>1 it will always be greater than 1.80.

Now consider $V(\overline{DC})$. It was previously shown that

$$V'(\overline{DC})^{\frac{1}{2}} = \left[0.3588 - 0.1321b\right] E'(\overline{DC})$$

Hence, $$V(\overline{DC}) = \{\left[0.3588 - 0.1321b\right] E'(DC)\}^2 (\frac{L}{hv})^2$$

if the horizontal travel time is dominating.

Therefore,

$$V(\overline{DC}/b=1) = 0.2267 \times E'(DC/b=1)^2 (\frac{L}{hv})^2$$

and $$V(DC/b=c^{-2}) = \{\left[0.3588 - \frac{0.1321}{c^2}\right] E'(DC/b=c^{-2})\}c^2 \cdot (\frac{L}{hv})^2 \qquad (5\text{-}17)$$

For b=1, (5-16) gives $(0.2267 \times 1.80)^2 (\frac{L}{hv})^2 = 0.1665 (\frac{L}{hv})^2$ (5-18)

anf for L' = cL, i.e. $b = c^{-2}$, (5-17) gives

$$V(DC/b=c^{-2}) = \left[\left(\frac{4}{3} + \frac{1}{2c^4} - \frac{1}{30c^6}\right)^2 \left(0.3588 - \frac{0.1321}{6^2}\right)^2 \right] c^2 \left(\frac{L}{hv}\right)^2 \quad (5\text{-}19)$$

We have to show:

$$\left[\left(\frac{4}{3} + \frac{1}{2c^4} - \frac{1}{30c^6}\right)^2 \left(0.3588 - \frac{0.1321}{c^2}\right)^2 \right] c^2 \geq 0.1665 \text{ for } c \geq 1$$

The left-hand side of the above inequality is equal to 0.1665 at c=1. Calculations show that the function on the left-hand side increases as one further increases c.

In summary, it has been shown that, under randomized storage the expected single and dual command travel times and their corresponding variances are minimized at b=1. Now, consider the over-all expected travel time and its variance. Recall that, DUAL percent of the trips are dual cycles. Since expectation is a linear operator, one can conclude that the over-all expected mean travel time will be minimized at b=1. Variance is not a linear operator; therefore, it is difficult to demonstrate that the over-all variance is minimized at b=1. However, the following argument will motivate the point. Assuming that DUAL > 0, there are only two events that may occur: single cycle or dual cycle; and both have positive probabilities. But we have already shown that the variance is minimized both for single and dual cycle when b=1. Therefore, no matter which event occurs we know that the corresponding variance will be minimized if b=1. Hence, in the long-range, one would expect the over-all variance to be minimized when b=1.

Next consider dedicated storage. Closed form expressions are not available for mean travel time and its variance. However, there is probably no obvious reason to state that the conclusion reached for randomized storage will not be true for dedicated. In fact, consider Figure 5-2, where it is assumed that b=1. Taken individually, we know that the area corresponding to class I will have minimum expected travel time and variance because it is square-in-time. Now, consider the "total" area taken by classes I and II. If $(C/A)_1 = (C/A)_2$, then we know that, once again the expected travel time and variance will be minimized since the aforementioned total area is square-in-time. If $(C/A)_1 \neq (C/A)_2$, then the only difference is that, comparatively more stops are made in either area I or area II (depending on their relative c_i values); but the mean and variance of travel time is still minimized because both area I and area (I + II) are square-in-time. The argument for the entire rack is similar to that presented above, because the number of areas that are square-in-time will approach zero, as b approaches zero. To support the above discussion a computer program was written, where for given descending (C/A) ratios the program first allocates the product classes to the openings and then computes the mean and variance of the travel time for five different levels. If NOL_s denotes the number of levels that maximizes the value of b, then the previously mentioned five levels correspond to the range defined by $NOL_s \pm 2$. NOL_s is determined from the following expression:

$$NOL_s = \frac{[(G \times w \times h)/R]^{\frac{1}{2}}}{h} \qquad (5\text{-}20)$$

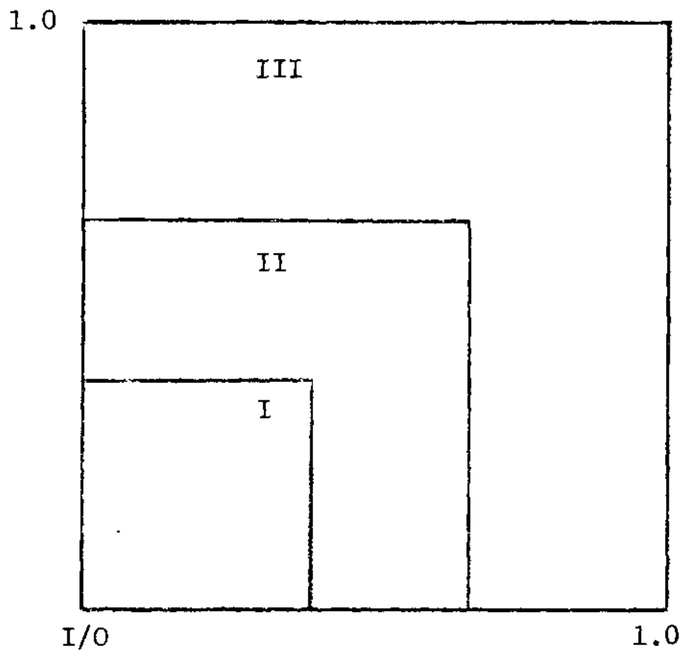


Figure 5-2. Allocation of three product classes over a rack that is square-in-time

where G = total number of openings on the rack

w = slot width

h = slot height

$R = hv/vv$

Ten different runs have been made with the above program. One of them is seen in Table 5-1. The rest are presented in Appendix 9. As seen from the printouts, the third level, i.e. NOL always minimizes the mean and variance of the travel time. (The program listing of the above program is presented in Appendix 10.

In summary, this section has shown that whichever storage method is used and whether waiting time is included or not, the throughput will be maximized when the rack is square-in-time. This result will be used in developing the algorithm which is presented in the following section.

Description of the Solution Procedure

First, recall the following:

λ = the required throughput level specified by the user.

TPUT(s)= the throughput level corresponding to s aisles.

h' = the number of levels that minimize h_2(NOL), where h_2(NOL) is the cost function corresponding to (total variable cost - S/R cost). Also, h' is determined from (5-8).

NOL_1 = the number of levels that minimize TC(NOL), where TC(NOL) is the total variable cost. NOL_1 is determined from (5-9).

NOL_s = the number of levels that maximize b (determined from (5-20)).

Also, let

$NOL(s)^*$ = the number of levels that minimize TC(NOL) <u>and</u> meet the throughput requirement, given s number of aisles.

Table 5-1. Sample Run Showing That Travel Time and Its Variance are Minimized at b=1

```
THE RACK IS SQUARE IN TIME FOR  7 LEVELS

NUMBER OF LEVELS=  5

SINGLE COMMAND(MEAN & VAR.)   .2168  .0150   DUAL COMMAND(MEAN & VAR.)   .3114  .0175

NUMBER OF LEVELS=  6

SINGLE COMMAND(MEAN & VAR.)   .2094  .0109   DUAL COMMAND(MEAN & VAR.)   .2976  .0115

NUMBER OF LEVELS=  7

SINGLE COMMAND(MEAN & VAR.)   .2074  .0098   DUAL COMMAND(MEAN & VAR.)   .2938  .0100

NUMBER OF LEVELS=  8

SINGLE COMMAND(MEAN & VAR.)   .2076  .0100   DUAL COMMAND(MEAN & VAR.)   .2942  .0101

NUMBER OF LEVELS=  9

SINGLE COMMAND(MEAN & VAR.)   .2103  .0114   DUAL COMMAND(MEAN & VAR.)   .2990  .0121
```

TC(s) = total cost of a system with s aisles.

The following algorithm is proposed:

1 - Set the upper limit on the number of aisles, call it M.

2 - Set NOAI = M and determine NOL_s. Then compute TPUT(M) for NOL = NOL_s. If TPUT(M) $\geq \lambda$, go to step 3. Otherwise, print an appropriate message and stop.

3 - Initiate the Fibonacci search over NOAI, with the initial interval of uncertainty being $[I,M]$.

4 - Set NOAI = F, where each F is dictated by the Fibonacci search.

5 - Determine NOL_s, and compute TPUT(F) for NOL = NOL_s. If TPUT(F) $\geq \lambda$, go to step 6. Otherwise, set TC(F) = ∞ and go to step 4.

6 - Determine h'. Then find NOL_1. Compute TPUT(F) for NOL = NOL_1. If TPUT(F) $\geq \lambda$, then set NOL(F)* = NOL_1 and return TC(F) with NOL = NOL(F)* to step 4. If TPUT(F) $< \lambda$, go to step 7.

7 - Determine the direction d, in which the throughput increases. (This direction will be unique because it was previously established that throughput increases as b approaches 1).

If $NOL_1 < NOL_s$, then set d = +1

If $NOL_1 > NOL_s$, then set d = -1

Set $NOL_d = NOL_1$ and go to step 8.

8 - Let $NOL_d = NOL_d + d$. Compute TPUT(F) for NOL = NOL_d. If TPUT(F) $\geq \lambda$, go to step 9. Otherwise, repeat this step

until throughput is satisfied (note: we are guaranteed that throughput will eventually be satisfied or alse we would have not been able to pass step 5).

9 - Set NOL(F)* = NOL_d and determine TC(F) with NOL = NOL(F)*. Return this cost value to step 4.

The algorithm stops when the Fibonacci search ends. Suppose the final uncertainty interval was $\left[F_1, F_2\right]$. The minimum cost design is then found by enumerating over the range F_1 to F_2.

One remark is concerned with the possibility of missing the global minimum through the Fibonacci search. It is difficult to establish the convexity of the total variable system cost in terms of NOAI and NOL, where both are allowed to vary simultaneously. The difficulty arises from the piecewise nature of the S/R cost, which is a function of both NOAI and NOL. However, based on empirical findings, there is very strong evidence that the total variable system cost is a convex function of NOAI and NOL. In all of the runs, where the program was forced to enumerate over the entire range $\left[I, M\right]$, there was no indication of local mimina.

The flow-chart of the above algorithm is presented in Figure 5-3. The corresponding program listing is given in Appendix 11. The functional definitions of the subroutines are given throughout the program listing.

Summary

The mathematical model of the system has been presented in terms of the design variables. The complexity of the throughput constraint and the non-linearity of the model prevents one from using classical

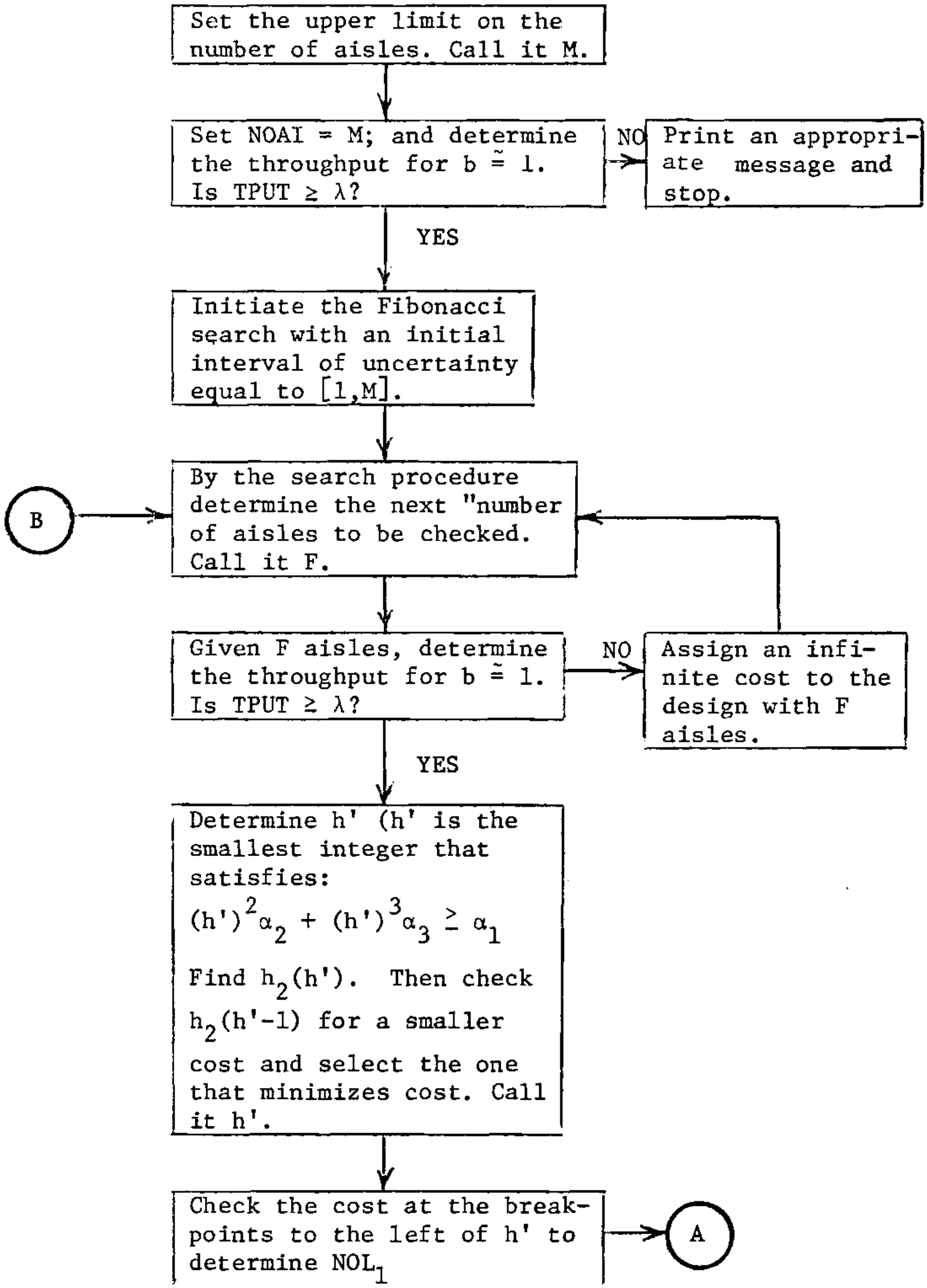
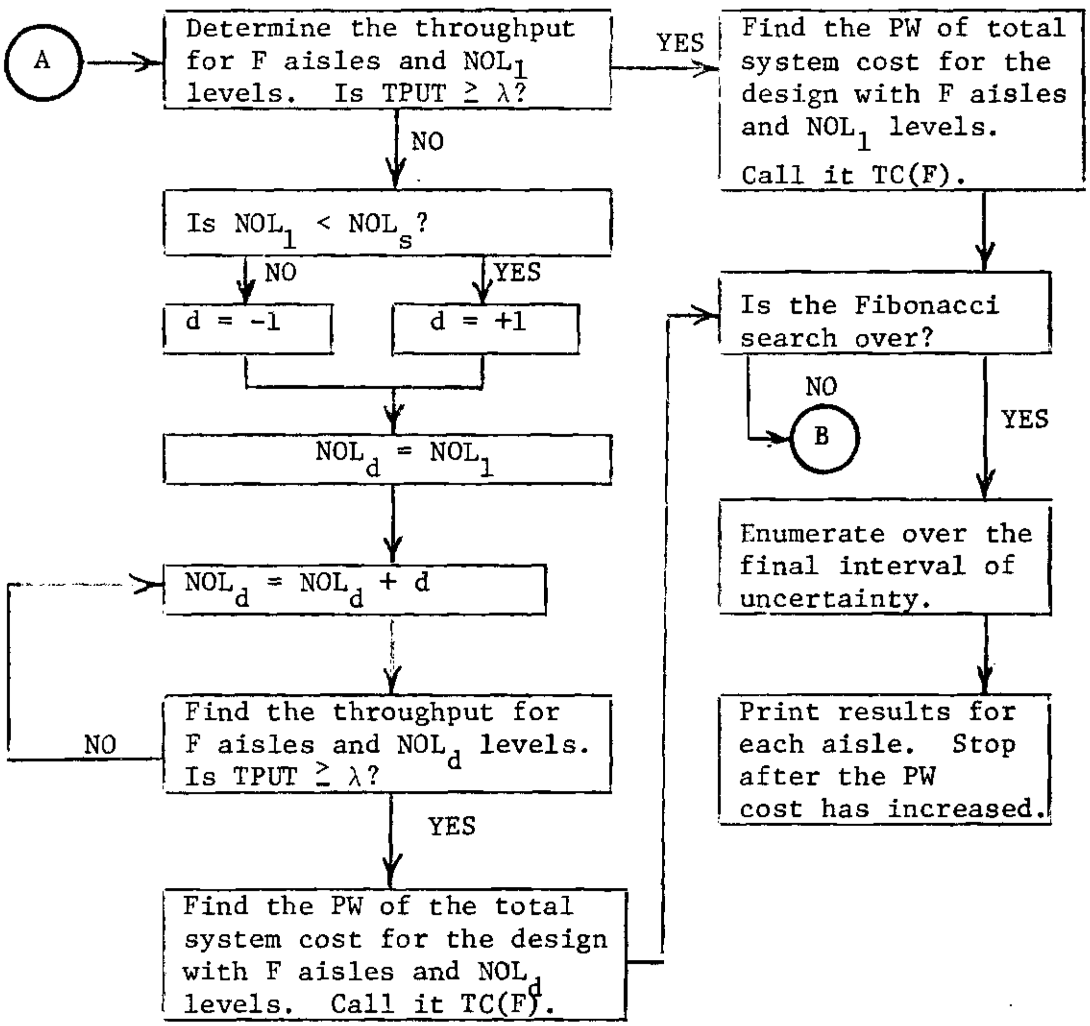


Figure 5-3. Flow-Chart of the Optimization Algorithm


A
Determine the throughput for F aisles and NOL_1 levels. Is TPUT ≥ λ?
YES
Find the PW of total system cost for the design with F aisles and NOL_1 levels. Call it TC(F).
NO
Is NOL_1 < NOL_s?
NO
YES
d = -1
d = +1
NOL_d = NOL_1
NOL_d = NOL_d + d
Find the throughput for F aisles and NOL_d levels. Is TPUT ≥ λ?
NO
YES
Find the PW of the total system cost for the design with F aisles and NOL_d levels. Call it TC(F).
Is the Fibonacci search over?
NO
B
YES
Enumerate over the final interval of uncertainty.
Print results for each aisle. Stop after the PW cost has increased.

optimization techniques. The analysis of cost elements, however, indicates that when the number of aisles is fixed, the variable cost elements (except S/R cost) are convex functions of the number of levels. Also, it is established that throughput is maximized when the rack is square-in-time. Hence, based on the results of the above two findings, it was possible to develop an algorithm that finds the minimum cost design with a corresponding feasible throughput level.

CHAPTER VI

DISCUSSION OF SAMPLE RUNS AND SENSITIVITY ANALYSIS

Introduction

Four sample runs will be presented and discussed in terms of cost minimization. Furthermore, a sensitivity analysis will be performed on two model parameters, namely "DUAL" and "λ".

Sample Runs

The program presented in Appendix 11 was executed with the data set shown in Figure 6-1. Inspection of the data indicates a high throughput required from the system (310 operations/hr). The solution to this data set is shown in Figure 6-2. Note that, enumeration over the final interval of uncertainty does not continue when present worth cost increases. In reference to Figure 6-2, since 611,502.59 > 576,560.58 the program stops after 8 aisles. Also, it can be seen that for 7 aisles, the system achieves a throughput of 330.50 operations/hr, which is greater than 310. At this point, one may argue that, for any data set the optimum number of aisles will always be the smallest number of aisles that meet the throughput constraint and therefore adding additional aisles will always cause the cost to increase while providing additional throughput capacity. The above argument may look intuitively correct, but it is not true. Consider the data set shown in Figure 6-3. The

```
GEORGIA TECH. - DESIGN OF AN AS/RS

  TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 30. 48. 30.
  TYPE UNIT LOAD WEIGHT(LB.S)
? 800.
  TYPE DUAL COMMAND PERCENTAGE
? 0.3
  TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 61. 250.
  TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.25 0.25
  TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 22.0 0.40
  TYPE 0,1,2 OR 3 FOR S/R LOGIC
    0=MAN-ON-BOARD  1=ON THE S/R
    2=OFF THE S/R  3=CENTRAL CONSOLE
? 1
  TYPE ANNUAL MAINTENANCE COST/SR
? 150.
  TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 1.5
  TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 0
  TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 1
  TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.1 0.5
  TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 0
  TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)
? 5000.
  TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)
? 310.
```

Figure 6-1. Data of Sample Run Number 1

```
NO. OF AISLES     7
NO. OF LEVELS       9.00
NO. OF COLUMNS      40.00

BLDG. LENGTH     220.67
BLDG. WIDTH      63.00
BLDG. HEIGHT      34.00

LAND COST    5560.00
BLDG. COST 25[illegible].25
RACK COST 281924.06
S/R MACHINE COST 364000.00   DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST     20853.00
S/R MAINTENANCE COST      1050.00

SYSTEM THROUGHPUT(0)      18.08 OP.NS/HR.
SYSTEM THROUGHPUT(1)     330.50 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     1.35   [illegible]
THE SHAPE FACTOR IS        .66

TOTAL NO. OF OPENINGS   5040.00


PW COST OF ABOVE DESIGN IS     5[illegible]0.50 DOLLARS
(TAX LIABILITY SHOULD BE   1[illegible]9.28)



NO. OF AISLES     8
NO. OF LEVELS       9.00
NO. OF COLUMNS      35.00

BLDG. LENGTH     197.33
BLDG. WIDTH      72.00
BLDG. HEIGHT      34.00

LAND COST    5683.20
BLDG. COST 263752.73
RACK COST 281924.06
S/R MACHINE COST 416000.00   DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST     [illegible]
S/R MAINTENANCE COST      1200.00

SYSTEM THROUGHPUT(0)      81.04 OP.NS/HR.
SYSTEM THROUGHPUT(1)     399.75 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     1.20   1.05
THE SHAPE FACTOR IS        .75

TOTAL NO. OF OPENINGS   5040.00


PW COST OF ABOVE DESIGN IS     611502.59 DOLLARS
(TAX LIABILITY SHOULD BE > 146535.36)


   .157 CP SECONDS EXECUTION TIME
```

Figure 6-2. Solution to Sample Run Number 1

data tabulated in Figure 6-3 is identical to the data set previously given in Figure 6-1 with the exception that the required throughput level has been increased to 340 operations/hr. The solution is shown in Figure 6-4. Note that, for this case, while the number of aisles has increased from 7 to 8, the cost has decreased from 629,237.77 to 611,502.59 dollars. Furthermore, the throughput increases from 340.80 to 399.75 operations/hr. That is, it was possible to achieve a higher throughput level with a lower cost. The result can be explained as follows: in the solution to the first run (Figure 6-2) note that both 7 and 8 aisle designs have 9 levels, hence by adding an aisle we have not gained much in terms of cost savings; in fact, we have incurred the cost of an additional S/R (note the sharp increase in S/R cost vs. slight changes in the other cost elements). Now suppose we increase the throughput requirement to 340 operations/hr. To meet the new throughput level, the design with 7 aisles now has to move towards a rack that is square-in-time. Figure 6-4 shows that for a 7 aisle design, the rack becomes square at 11 levels (note that the shape factor is 0.98). Hence, we are still able to meet the required throughput with 7 aisles, but in doing so, we forced the 7 aisle design to move further away from the constraint-free design. Now consider 8 aisles. The 8 aisle design has a throughput capacity of 399.75 operations/hr in Figure 6-2. Therefore, increasing the required throughput will not change the answer to the 8 aisle design (note that the 8 aisle design is identical in Figures 6-2 and 6-4). Hence, by adding an aisle, we were able to arrive to a design that comes closer to its constraint-free design and thus provides a

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  TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 30. 48. 30.
  TYPE UNIT LOAD WEIGHT(LB.S)
? 800.
  TYPE DUAL COMMAND PERCENTAGE
? 0.3
  TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 61. 250.
  TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.25 0.25
  TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 22.0 0.40
  TYPE 0,1,2 OR 3 FOR S/R LOGIC
    0=MAN-ON-BOARD  1=ON THE S/R
    2=OFF THE S/R  3=CENTRAL CONSOLE
? 1
  TYPE ANNUAL MAINTENANCE COST/SR
? 150.
  TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 1.5
  TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 0
  TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 1
  TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.1 0.5
  TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 0
  TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)
? 5000.
  TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)
? 340.
```

Figure 6-3. Data of Sample Run Number 2

```
NO. OF AISLES      7
NO. OF LEVELS       11.00
NO. OF COLUMNS       33.00

BLDG. LENGTH     188.00
BLDG. WIDTH      63.00
BLDG. HEIGHT      40.67

LAND COST    4737.60
BLDG. COST 245570.74
RACK COST 317468.24
S/R MACHINE COST 455000.00   DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST     17766.00
S/R MAINTENANCE COST      1050.00

SYSTEM THROUGHPUT(0)          .71 OP.NS/HR.
SYSTEM THROUGHPUT(1)      340.80 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     1.31    2.10
THE SHAPE FACTOR IS          .98

TOTAL NO. OF OPENINGS    5082.00


PW COST OF ABOVE DESIGN IS     629237.77 DOLLARS
(TAX LIABILITY SHOULD BE > 169407.80)




NO. OF AISLES      8
NO. OF LEVELS        9.00
NO. OF COLUMNS       35.00

BLDG. LENGTH     197.33
BLDG. WIDTH      72.00
BLDG. HEIGHT      34.00

LAND COST    5683.20
BLDG. COST 263752.73
RACK COST 281924.06
S/R MACHINE COST 416000.00   DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST     21312.00
S/R MAINTENANCE COST      1200.00

SYSTEM THROUGHPUT(0)       53.45 OP.NS/HR.
SYSTEM THROUGHPUT(1)      399.75 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     1.28    2.05
THE SHAPE FACTOR IS         .75

TOTAL NO. OF OPENINGS    5040.00


PW COST OF ABOVE DESIGN IS     611502.59 DOLLARS
(TAX LIABILITY SHOULD BE > 146535.36)
```

Figure 6-4. Solution to Sample Run Number 2

```
NO. OF AISLES     9
NO. OF LEVELS      9.00
NO. OF COLUMNS     31.00

BLDG. LENGTH     178.67
BLDG. WIDTH     81.00
BLDG. HEIGHT     34.00

LAND COST    5788.80
BLDG. COST 268653.54
RACK COST 280917.19
S/R MACHINE COST 466000.00    DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST     11708.00
S/R MAINTENANCE COST      1559.00

SYSTEM THROUGHPUT(0)     118.77 OP.NO./HR.
SYSTEM THROUGHPUT(1)     470.43 OP.NO./HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     1.22    1.
THE SHAPE FACTOR IS         .85

TOTAL NO. OF OPENINGS    5022.00

PW COST OF ABOVE DESIGN IS     645259.01 DOLLARS
(TAX LIABILITY SHOULD BE ) 155114.15)


   .184 CP SECONDS EXECUTION TIME
```

smaller cost. It is also instructive to note the S/R costs for 7 and 8 aisles in Figure 6-4. It is seen that the S/R cost is 455,000 and 416,000 dollars for 7 and 8 aisles, respectively. That is, though we purchased an additional S/R for the 8 aisle design, the "total" S/R cost has decreased. This reduction in total S/R cost is due to a high cost per S/R in the 7 aisle design. (Recall that S/R cost is a function of the number of levels). Also note that, since the present worth cost decreased, the program does not stop and carries the search over to 9 aisles.

The third sample run demonstrates a man-on-board design. The data set is presented in Figure 6-5. Note that, for the S/R logic equal to 0, the program asks the annual operator cost. The solution is shown in Figure 6-6. The additional labor cost is printed under "total operator cost".

The fourth sample run shows the dedicated storage case. The data set is presented in Figure 6-7. Note that, as opposed to sample runs presented previously, the number of openings and the throughput level are determined from the A_i and c_i values entered respectively by the user. Furthermore, the user has the freedom to choose between simulation and enumeration for dual cycle travel time computation. The solution is shown in Figure 6-8. The number of openings for each product class are given on a per rack basis.

Sensitivity Analysis

There are many parameters that stand as candidates for sensitivity analysis, but among them only two require considerable judgement and/or

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   TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 48. 48. 48.
   TYPE UNIT LOAD WEIGHT(LB.S)
? 2000.
   TYPE DUAL COMMAND PERCENTAGE
? 0.5
   TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 60. 240.
   TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.25 0.25
   TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 25. 6.
   TYPE 0,1,2 OR 3 FOR S/R LOGIC
     0=MAN-ON-BOARD  1=ON THE S/R
     2=OFF THE S/R  3=CENTRAL CONSOLE
? 0
   TYPE ANNUAL COST/OPERATOR
? 20000.
   TYPE ANNUAL MAINTENANCE COST/SR
? 2200.
   TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 5.5
   TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 0
   TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 1
   TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.10 0.50
   TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 0
   TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)
? 6000.
   TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)
? 150.
```

Figure 6-5. Data of Sample Run Number 3

```
NO. OF AISLES      5
NO. OF LEVELS       11.00
NO. OF COLUMNS       55.00

BLDG. LENGTH      290.67
BLDG. WIDTH       67.50
BLDG. HEIGHT       57.17

LAND COST 117720.00
BLDG. COST 640655.54
RACK COST 502036.82
S/R MACHINE COST 390000.00    DOLLARS

THE RECURRING COSTS ARE

BLDG..MAINTENANCE COST    107910.00
S/R MAINTENANCE COST     11000.00
TOTAL OPERATOR COST   100000.00 DOLLARS/YR.

SYSTEM THROUGHPUT(0)      31.68 OP.NS/HR.
SYSTEM THROUGHPUT(1)     187.47 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     1.81    2.77
THE SHAPE FACTOR IS         .83

TOTAL NO. OF OPENINGS    6050.00


PW COST OF ABOVE DESIGN IS    1594757.29 DOLLARS
(TAX LIABILITY SHOULD BE > 276638.47)



NO. OF AISLES      6
NO. OF LEVELS       11.00
NO. OF COLUMNS       46.00

BLDG. LENGTH      248.67
BLDG. WIDTH       81.00
BLDG. HEIGHT       57.17

LAND COST 120852.00
BLDG. COST 665913.35
RACK COST 503862.41
S/R MACHINE COST 468000.00    DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST    110781.00
S/R MAINTENANCE COST     13200.00
TOTAL OPERATOR COST   120000.00 DOLLARS/YR.

SYSTEM THROUGHPUT(0)      79.77 OP.NS/HR.
SYSTEM THROUGHPUT(1)     240.94 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     1.69    2.60
THE SHAPE FACTOR IS         .99

TOTAL NO. OF OPENINGS    6072.00


PW COST OF ABOVE DESIGN IS    1727704.90 DOLLARS
(TAX LIABILITY SHOULD BE > 294755.15)


   .182 CP SECONDS EXECUTION TIME
```

Figure 6-6. Solution to Sample Run Number 3

```
GEORGIA TECH. - DESIGN OF AN AS/RS

  TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 48. 46. 46.
  TYPE UNIT LOAD WEIGHT(LB.S)
? 1300.0
  TYPE DUAL COMMAND PERCENTAGE
? 0.40
  TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 55. 230.
  TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.20 0.20
  TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 23.0 3.5
  TYPE 0,1,2 OR 3 FOR S/R LOGIC
    0=MAN-ON-BOARD  1=ON THE S/R
    2=OFF THE S/R  3=CENTRAL CONSOLE
? 3
  TYPE ANNUAL MAINTENANCE COST/SR
? 1800.
  TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 5.5
  TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 1
  TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 0
  TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.10 0.50
  TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 1
  TYPE NO. OF PRODUCT CLASSES(INTEGER)
? 2
  TYPE OP.N/HR,NO. OF SLOTS FOR EACH PROD. CLASS
? 60. 2000
? 50. 2100
  FOR ENUMERATION TYPE 1,FOR SIMULATION TYPE 0
? 0
```

Figure 6-7. Data of Sample Run Number 4

```
NO. OF AISLES      6
NO. OF LEVELS        9.00
NO. OF COLUMNS      38.00

BLDG. LENGTH     205.00
BLDG. WIDTH      83.00
BLDG. HEIGHT      46.00

LAND COST  59552.50
BLDG. COST 513170.58
RACK COST 294756.85
S/R MACHINE COST 624000.00    DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST      93582.50
S/R MAINTENANCE COST     10800.00

SYSTEM THROUGHPUT(0)      169.38 OP.NS/HR.
SYSTEM THROUGHPUT(1)      290.96 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     1.37    2.08
THE SHAPE FACTOR IS         .98

   167.00 OPENINGS FOR PROD. CLASS    1
   175.00 OPENINGS FOR PROD. CLASS    2


PW COST OF ABOVE DESIGN IS    1208991.24 DOLLARS
(TAX LIABILITY SHOULD BE > 127551.37)




NO. OF AISLES      7
NO. OF LEVELS        9.00
NO. OF COLUMNS      33.00

BLDG. LENGTH     182.50
BLDG. WIDTH      96.50
BLDG. HEIGHT      46.00

LAND COST  61639.38
BLDG. COST 531174.12
RACK COST 298635.23
S/R MACHINE COST 728000.00    DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST      96861.88
S/R MAINTENANCE COST     12600.00

SYSTEM THROUGHPUT(0)      232.95 OP.NS/HR.
SYSTEM THROUGHPUT(1)      355.98 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     1.31    1.98
THE SHAPE FACTOR IS         .88

   143.00 OPENINGS FOR PROD. CLASS    1
   150.00 OPENINGS FOR PROD. CLASS    2


PW COST OF ABOVE DESIGN IS    1299366.29 DOLLARS
(TAX LIABILITY SHOULD BE > 143927.05)


 33.250 CP SECONDS EXECUTION TIME
```

Figure 6-8. Solution to Sample Run Number 4

past data for estimation. Those are, "DUAL" (indicates the percent of trips done on a dual command basis) and λ (the throughput level demanded from the system).

In general it is common practice to perform sensitivity analysis studies on optimization programs that require many and/or difficult parameter estimations. However, in the context of the optimization algorithm developed in this study, it is not possible to give general statements about sensitivity because the importance (or effect) of a certain parameter will vary from problem to problem, depending on the relative locations of the constraint-free optimum and that optimum dictated by constraint involving the parameter of interest. For example, consider "DUAL". At any particular instance of a problem, if the optimum design has some "slack" throughput capacity, then, decreasing "DUAL" will not change the answer, and hence one will be tempted to conclude that the minimum cost is not sensitive to "DUAL". However, imagine an optimum design in which there is virtually no slack throughput capacity. Then, reducing "DUAL" will force the design and the corresponding cost to change immediately. This argument is supported by the results tabulated in Table 6-1. (The computer printouts of Table 6-1 can be seen in Appendix 12). Note that, different conclusions can be made in terms of sensitivity over different ranges of "DUAL". Hence, it is strongly recommended that every user should perform their own sensitivity analysis over a particular data set.

Next consider the required throughput, λ. The above argument given for "DUAL" holds for this parameter also. Table 6-2 presents the

TABLE 6-1 SENSITIVITY ANALYSIS RESULTS FOR "DUAL"

DUAL	PW	NOAI	TPUT
0.0	1,365,464.72	5	165.36
0.2	1,365,464.72	5	173.55
0.4	1,365,464.72	5	182.59
0.6	1,365,464.72	5	192.62
0.8	1,285,264.65	4	146.81
1.0	1,285,264.65	4	156.42

TABLE 6-2. SENSITIVITY ANALYSIS RESULTS FOR THE REQUIRED THROUGHPUT LEVEL

TPUT	PW	NOAI
100	1,285,264.65	4
125	1,285,264.65	4
150	1,365,464.72	5
200	1,452,553.83	6

the results obtained from four different throughput levels (denoted TPUT in the table). Note that, even we have mentioned the hazards of making general conclusions, Table 6-2 indicates that present worth cost is fairly insensitive to λ (TPUT).

Another remark is concerned with the waiting time associated to each operation. It would be recalled that in computing the system throughput, the user has the option of including "WT", where "WT" denotes the time spent by an order waiting in the queue. Hence, if the user has required that "WT" should be included in the throughput calculation, then in the computer printout, the throughput level given under the heading "system throughput (0)" should be greater than or equal to λ. (Obviously, "system throughput (1)" will always be greater than "system throughput (0)", because (1) denotes that "WT" is not considered in computing the throughput level of the system). It should also be noted that the expression used for WT assumed a $(M/G/1):(GD/\infty/\infty)$ queue.

From previous sample runs note that in a given run, the difference between "system throughput (1)" and "system throughput (0)" is always larger for the initially printed number of aisles compared to the difference found from the following design which has an additional aisle. This is due to the fact that as the number of aisles decreases, λ' increases. (Recall from queueing theory that the waiting time will exponentially increase as the arrival rate increases).

Summary

Four sample runs were presented and discussed. It was demonstrated that it is possible to increase throughput while decreasing cost. In addition, in the light of the sensitivity analysis, it can be concluded that total cost seems to be fairly insensitive to the required throughput level. Also, the degree of importance of each parameter will vary with the nature of the optimum design.

CHAPTER VII

CONCLUSION AND RECOMMENDATIONS

Introduction

In analyzing AS/R systems and developing the optimization model for minimum cost AS/RS design, a number of insights were obtained. In this chapter, the conclusions reached during the study are provided and recommendations for further study are presented.

Conclusions

The conclusions reached in the conduct of the study can be stated as follows:

1) The relative space requirement for randomized and dedicated storage methods is a function of two variables: the average difference between the total number of parts received in two consecutive time periods and the average number of lost sales. (Note that, additional variables may have to be considered if a different inventory model is used). Simulation results obtained showed that, under the <R,r,T> policy, dedicated storage may require 20% to 60% more space than randomized.

2) It is possible to reduce the mean S/R travel time by using dedicated storage instead of randomized. Graves, et.al. (7) and Barrett (3) report various ranges of reduction depending upon the spread of the

turnover on each pallet. However, computational experience has shown that the expected variance of travel time is not necessarily reduced when dedicated storage is used. In fact, there have been some cases where the expected variance increased when DUAL $\geq$ 0.80. Hence, if WT (waiting time) is included in the throughput calculation, the expected throughput level may not significantly increase when one uses dedicated storage instead of randomized.

3) Whether waiting time is included or not, throughput will be maximized when b=1, i.e. when the rack is square-in-sime. (A similar conclusion is also presented by Bafna (2)). However, when design costs are included, the optimum design, will not necessarily be square-in-time.

4) Under randomized storage the single command mean travel time follows a theoretical distribution presented in Chapter IV; the dual command mean travel time is Beta distributed.

5) When the number of aisles is fixed, the variable cost elements constitute a piece-wise convex function in the number of levels. The break-points are those number of levels that correspond to the height intervals defined under the S/R cost.

6) By adding an aisle to a given design, in some cases it may be possible to increase the throughput while decreasing the total present worth cost of the system.

7) It is not correct to make general conclusions regarding the sensitivity of total system cost to "DUAL". The importance of "DUAL"

will depend upon the particular design under analysis. On the other hand, sample runs show that the total system cost is fairly insensitive to the required throughput.

8) The waiting time grows exponentially when the arrival rate per aisle is increased. Hence, systems with a small number of aisles will always show a larger difference in throughput levels obtained by including and not including waiting time.

9) The conventional method of computing travel time presented in Chapter IV will always underestimate single command mean travel time. It may underestimate or overestimate the dual command mean travel time.

Recommendations

A number of areas for further study were identified in performing the study. Areas for further study felt to be significant due to either their theoretical contribution or their usefulness in designing real world systems include the following:

1) Consider transfer cars for systems with low throughput requirements and/or for systems where every item is not carried in every aisle. One alternative would be to consider the S/R's and the transfer cars within a network of queues and use simulation to analyze system behavior and draw conclusions.

2) Develop closed form expressions for the expected mean and variance of the single command and dual command travel times under dedicated storage for fixed and/or variable number of product classes.

3) Further analyze the effect of square-L boundaries versus

concentric squares on "travel between" and round-trip time. Extend the work done by Graves, et.al. in (6) and (7).

4) Develop an analytical approach to establish the proper balance between storage capacity and system throughput. Constract a mathematical relation between the two in terms of system parameters and/or variables.

5) Extend the present analysis to consider multiple sized, vertical openings in the rack and multiple part numbers on a pallet. Analyze the effect of multi-level I/O stations on throughput. Analyze the effect of the vertical location of the I/O station on throughput and cost.

6) Extend the analysis of AS/RS configurations to include mini-load systems. Develop appropriate cost models and consider multiple part numbers per storage bin.

7) Develop analytical and/or simulation models for carousel S/R systems. Analyze the effects on throughput of centralized versus decentralized storage and retrieval and randomized versus dedicated storage.

8) Extend the present analysis to include deep lane storage systems, as well as multi-depth AS/R systems.

9) Analyze the economic impact of increasing S/R travel velocities in order to meet the throughput requirement over a given rack.

10) Analyze the effect of deviating from FCFS on retrievals. Compare the advantage of achieving lower dual command trip times against the cost of a more sophisticated software.

11) Analyze the travel time for the "pure random" and "closest-open location" storage methods. Determine the amount of overestimation in travel time caused by using the pure random approach.

12) Update the current program to reflect changes in the tax laws for 1979.

APPENDIX 1

Program listing for simulating the storage space requirement under dedicated and randomized storage

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION NOR(100,260),INV(100),IR(100),LS(100)
      DIMENSION IRT(100),MCUM(100),MON(100),MAX(100),ID(100)
      DIMENSION DEM(100),MC(16),IAMP(16),RLAM(16)
      COMMON RLAM,DEM,IAMP,MC
      DO 2 K7=1,9
      DEM(K7)=20.0
      IRT(K7)=62
2     IR(K7)=44
      DO 3 K8=10,24
      DEM(K8)=12.0
      IRT(K8)=46
3     IR(K8)=31
      DO 4 K9=25,84
      DEM(K9)=4.0
      IRT(K9)=17
4     IR(K9)=7
      DO 5 K6=1,16
5     READ(5,*)IAMP(K6),RLAM(K6),MC(K6)
      DO 33 K14=85,94
      DEM(K14)=12.0
      IR(K14)=50
33    IRT(K14)=60
      DO 34 K15=95,100
      DEM(K15)=20.0
      IR(K15)=68
34    IRT(K15)=85
      LT=1
      IGT=0
      DO 10 K1=1,100
      INV(K1)=IRT(K1)
      MCUM(K1)=0
      MON(K1)=0
      LS(K1)=0
10    MAX(K1)=0
      CUDF=0.0
      RMX=0.0
      DO 999 KL=1,260
      ITOT=0
      DIFF=0.0
      CALL DGEN(ID,KL)
      DO 888 K=1,100
      INV(K)=INV(K)-ID(K)
      IF(INV(K))13,14,14
13    LS(K)=LS(K)-INV(K)
      IF(KL.LE.52)LS(K)=0
      INV(K)=0
14    IN=KL-LT
      IF(IN.LE.0)GO TO 15
      INV(K)=INV(K)+NOR(K,IN)
      DIFF=DIFF+NOR(K,IN)
      MON(K)=MON(K)-NOR(K,IN)
15    IPO=INV(K)+MON(K)
      IF(IPO-IR(K))19,19,20
19    NOR(K,KL)=IRT(K)-IPO
      MON(K)=MON(K)+NOR(K,KL)
      GO TO 17
20    NOR(K,KL)=0
17    IF(KL.LE.52)GO TO 888
```

```
      MCUM(K)=MCUM(K)+INV(K)
      ITOT=ITOT+INV(K)
      IF(INV(K).GT.MAX(K))MAX(K)=INV(K)
888   CONTINUE
      IF(ITOT.GT.IGT)IGT=ITOT
      IF(KL.LE.52)GO TO 998
      RR=ABS(DIFF-TD)
      CUDF=CUDF+RR
      IF(RR.GT.RMX)RMX=RR
998   TD=DIFF
999   CONTINUE
      RINVEN=0.0
      IDS=0
      ISAY=0
      DO 21 J1=1,100
      AV=MCUM(J1)/208.0
      RINVEN=RINVEN+AV
      IDS=IDS+MAX(J1)
      ISAY=ISAY+LS(J1)
21    WRITE(6,23)AV,LS(J1),J1
23    FORMAT(3X,'AV. INT.',2X,F9.4,2X,'L. SALES',2X,I5,2X,I3)
      WRITE(6,24)IGT,IDS
24    FORMAT(/3X,'RANDOMIZED',2X,I9,2X,'DEDICATED',2X,I9)
      AVER=CUDF/208.0
      RINVEN=RINVEN/100.0
      WRITE(6,74)AVER,RMX
74    FORMAT(/3X,'AV.DIFF.',2X,F10.4,2X,'MAX.DIFF.',2X,F10.4)
      WRITE(6,75)ISAY,RINVEN
75    FORMAT(3X,'TOT. LOST SALES',2X,I9,2X,'AV. AGG. INV.',2X,F10.4)
      STOP
      END
      SUBROUTINE DGEN(ID,KL)
      DIMENSION ID(100)
      DIMENSION DEM(100),IAMP(16),RLAM(16),MC(16)
      COMMON RLAM,DEM,IAMP,MC
      DO 11 K1=1,84
      CALL POS(IAR,DEM(K1))
11    ID(K1)=IAR
      DO 12 K2=1,16
      IW=84+K2
      A=IAMP(K2)*SIN((6.283/MC(K2))*(KL+RLAM(K2)))
      ARR=DEM(IW)+A
      CALL POS(IAR,ARR)
12    ID(IW)=IAR
      RETURN
      END
      SUBROUTINE POS(N,AL)
      I=1
      S=1.0
      P=1.0/EXP(AL)
2     S=S*RANF(X)
      IF(S.LE.P)GO TO 5
      I=I+1
      GO TO 2
5     N=I-1
      RETURN
      END
```

APPENDIX 2

Output of the simulation runs obtained from

the program listing in Appendix 1

AV. INT.	35.2933	L. SALES	9	1
AV. INT.	37.8894	L. SALES	5	2
AV. INT.	34.6442	L. SALES	21	3
AV. INT.	35.4856	L. SALES	35	4
AV. INT.	34.6250	L. SALES	34	5
AV. INT.	34.4856	L. SALES	12	6
AV. INT.	34.3990	L. SALES	13	7
AV. INT.	34.6058	L. SALES	23	8
AV. INT.	34.5962	L. SALES	21	9
AV. INT.	28.3173	L. SALES	14	10
AV. INT.	28.8269	L. SALES	11	11
AV. INT.	29.8702	L. SALES	0	12
AV. INT.	28.3317	L. SALES	2	13
AV. INT.	29.2788	L. SALES	15	14
AV. INT.	27.9087	L. SALES	1	15
AV. INT.	29.8029	L. SALES	3	16
AV. INT.	29.7452	L. SALES	12	17
AV. INT.	30.4808	L. SALES	3	18
AV. INT.	29.1394	L. SALES	5	19
AV. INT.	28.9087	L. SALES	4	20
AV. INT.	29.2067	L. SALES	7	21
AV. INT.	28.8846	L. SALES	5	22
AV. INT.	29.4567	L. SALES	7	23
AV. INT.	28.2452	L. SALES	2	24
AV. INT.	17.9712	L. SALES	0	25
AV. INT.	18.1442	L. SALES	0	26
AV. INT.	18.7692	L. SALES	0	27
AV. INT.	18.9183	L. SALES	0	28
AV. INT.	18.0481	L. SALES	0	29
AV. INT.	18.6394	L. SALES	0	30
AV. INT.	18.4327	L. SALES	0	31
AV. INT.	18.4087	L. SALES	0	32
AV. INT.	18.0721	L. SALES	1	33
AV. INT.	18.1250	L. SALES	0	34
AV. INT.	18.5337	L. SALES	0	35
AV. INT.	18.5913	L. SALES	0	36
AV. INT.	17.7981	L. SALES	0	37
AV. INT.	17.8654	L. SALES	0	38
AV. INT.	18.5577	L. SALES	0	39
AV. INT.	17.9712	L. SALES	0	40
AV. INT.	18.1010	L. SALES	0	41
AV. INT.	17.6298	L. SALES	0	42
AV. INT.	18.6779	L. SALES	0	43
AV. INT.	18.0240	L. SALES	0	44
AV. INT.	18.3942	L. SALES	0	45
AV. INT.	17.4663	L. SALES	0	46
AV. INT.	18.5144	L. SALES	6	47
AV. INT.	17.9904	L. SALES	0	48
AV. INT.	18.1106	L. SALES	0	49
AV. INT.	18.9423	L. SALES	0	50
AV. INT.	18.8221	L. SALES	0	51
AV. INT.	18.6106	L. SALES	0	52
AV. INT.	18.5096	L. SALES	0	53
AV. INT.	19.2212	L. SALES	0	54
AV. INT.	18.4327	L. SALES	0	55
AV. INT.	18.8365	L. SALES	0	56
AV. INT.	17.8462	L. SALES	0	57
AV. INT.	18.9471	L. SALES	0	58
AV. INT.	18.3173	L. SALES	0	59
AV. INT.	17.9712	L. SALES	0	60
AV. INT.	17.2452	L. SALES	0	61
AV. INT.	17.7500	L. SALES	0	62
AV. INT.	17.8654	L. SALES	0	63
AV. INT.	18.5000	L. SALES	0	64
AV. INT.	18.1779	L. SALES	0	65
AV. INT.	18.5385	L. SALES	0	66
AV. INT.	18.5433	L. SALES	0	67
AV. INT.	18.2067	L. SALES	0	68
AV. INT.	18.3942	L. SALES	0	69
AV. INT.	18.2019	L. SALES	0	70
AV. INT.	18.0625	L. SALES	0	71
AV. INT.	18.3654	L. SALES	0	72
AV. INT.	18.1490	L. SALES	0	73
AV. INT.	19.0240	L. SALES	0	74
AV. INT.	18.4808	L. SALES	0	75
AV. INT.	18.1635	L. SALES	0	76
AV. INT.	17.9760	L. SALES	0	77
AV. INT.	19.2019	L. SALES	0	78
AV. INT.	18.6923	L. SALES	0	79
AV. INT.	18.7067	L. SALES	0	80
AV. INT.	17.6298	L. SALES	0	81
AV. INT.	17.8029	L. SALES	0	82
AV. INT.	18.8173	L. SALES	0	83
AV. INT.	18.6538	L. SALES	0	84
AV. INT.	49.9808	L. SALES	5	85
AV. INT.	49.7067	L. SALES	7	86
AV. INT.	49.2548	L. SALES	0	87
AV. INT.	49.0192	L. SALES	0	88
AV. INT.	49.6106	L. SALES	2	89
AV. INT.	50.0817	L. SALES	0	90
AV. INT.	48.5913	L. SALES	0	91
AV. INT.	47.8269	L. SALES	3	92
AV. INT.	48.8317	L. SALES	3	93
AV. INT.	48.4712	L. SALES	0	94
AV. INT.	56.7019	L. SALES	7	95
AV. INT.	57.2596	L. SALES	43	96
AV. INT.	56.2308	L. SALES	50	97
AV. INT.	58.5096	L. SALES	11	98
AV. INT.	56.4904	L. SALES	26	99
AV. INT.	55.2452	L. SALES	27	100

RANDOMIZED 2876 DEDICATED 4104

AV.DIFF. 62.7260 MAX.DIFF. 229.0000
TOT. LOST SALES 455 AV. AGG. INV. 26.8360

AV.	INT.	32.6683	L. SALES	35	1
AV.	INT.	35.0048	L. SALES	13	2
AV.	INT.	31.9615	L. SALES	43	3
AV.	INT.	32.6875	L. SALES	49	4
AV.	INT.	32.0000	L. SALES	60	5
AV.	INT.	31.8894	L. SALES	40	6
AV.	INT.	31.6683	L. SALES	34	7
AV.	INT.	31.9231	L. SALES	45	8
AV.	INT.	31.8702	L. SALES	40	9
AV.	INT.	24.9327	L. SALES	35	10
AV.	INT.	25.5337	L. SALES	23	11
AV.	INT.	26.3990	L. SALES	10	12
AV.	INT.	25.0769	L. SALES	19	13
AV.	INT.	25.9038	L. SALES	31	14
AV.	INT.	24.6875	L. SALES	17	15
AV.	INT.	26.5577	L. SALES	17	16
AV.	INT.	26.5240	L. SALES	36	17
AV.	INT.	26.9471	L. SALES	20	18
AV.	INT.	25.6587	L. SALES	21	19
AV.	INT.	25.5913	L. SALES	18	20
AV.	INT.	26.0048	L. SALES	19	21
AV.	INT.	25.7115	L. SALES	41	22
AV.	INT.	26.1442	L. SALES	26	23
AV.	INT.	24.9760	L. SALES	31	24
AV.	INT.	15.9712	L. SALES	0	25
AV.	INT.	16.1587	L. SALES	1	26
AV.	INT.	16.7692	L. SALES	0	27
AV.	INT.	16.9183	L. SALES	0	28
AV.	INT.	16.0481	L. SALES	0	29
AV.	INT.	16.6394	L. SALES	0	30
AV.	INT.	16.4327	L. SALES	0	31
AV.	INT.	16.4375	L. SALES	2	32
AV.	INT.	16.1010	L. SALES	3	33
AV.	INT.	16.1394	L. SALES	1	34
AV.	INT.	16.5337	L. SALES	0	35
AV.	INT.	16.5913	L. SALES	0	36
AV.	INT.	15.8269	L. SALES	2	37
AV.	INT.	15.8654	L. SALES	0	38
AV.	INT.	16.5577	L. SALES	0	39
AV.	INT.	15.9712	L. SALES	0	40
AV.	INT.	16.1010	L. SALES	0	41
AV.	INT.	15.6298	L. SALES	0	42
AV.	INT.	16.6779	L. SALES	0	43
AV.	INT.	16.0240	L. SALES	0	44
AV.	INT.	16.4087	L. SALES	1	45
AV.	INT.	15.5096	L. SALES	3	46
AV.	INT.	16.5529	L. SALES	8	47
AV.	INT.	15.9904	L. SALES	0	48
AV.	INT.	16.1106	L. SALES	0	49
AV.	INT.	16.9423	L. SALES	0	50
AV.	INT.	16.8221	L. SALES	0	51
AV.	INT.	16.6106	L. SALES	0	52
AV.	INT.	16.5385	L. SALES	2	53
AV.	INT.	17.2212	L. SALES	0	54
AV.	INT.	16.4327	L. SALES	0	55
AV.	INT.	16.8365	L. SALES	0	56
AV.	INT.	15.8750	L. SALES	2	57
AV.	INT.	16.9760	L. SALES	2	58
AV.	INT.	16.3173	L. SALES	0	59
AV.	INT.	15.9712	L. SALES	0	60
AV.	INT.	15.2596	L. SALES	1	61
AV.	INT.	15.7500	L. SALES	0	62
AV.	INT.	15.9038	L. SALES	2	63
AV.	INT.	16.5000	L. SALES	0	64
AV.	INT.	16.1923	L. SALES	1	65
AV.	INT.	16.5385	L. SALES	0	66
AV.	INT.	16.5433	L. SALES	0	67
AV.	INT.	16.2212	L. SALES	1	68
AV.	INT.	16.4087	L. SALES	1	69
AV.	INT.	16.2308	L. SALES	2	70
AV.	INT.	16.0625	L. SALES	0	71
AV.	INT.	16.3654	L. SALES	0	72
AV.	INT.	16.1779	L. SALES	2	73
AV.	INT.	17.0240	L. SALES	0	74
AV.	INT.	16.4808	L. SALES	0	75
AV.	INT.	16.1779	L. SALES	1	76
AV.	INT.	15.9760	L. SALES	0	77
AV.	INT.	17.2019	L. SALES	0	78
AV.	INT.	16.6923	L. SALES	0	79
AV.	INT.	16.7067	L. SALES	0	80
AV.	INT.	15.6298	L. SALES	0	81
AV.	INT.	15.8317	L. SALES	2	82
AV.	INT.	16.8317	L. SALES	1	83
AV.	INT.	16.6538	L. SALES	0	84
AV.	INT.	50.4231	L. SALES	5	85
AV.	INT.	50.3173	L. SALES	6	86
AV.	INT.	49.6202	L. SALES	0	87
AV.	INT.	49.3125	L. SALES	0	88
AV.	INT.	50.1346	L. SALES	2	89
AV.	INT.	50.3798	L. SALES	0	90
AV.	INT.	48.8029	L. SALES	0	91
AV.	INT.	48.3510	L. SALES	3	92
AV.	INT.	49.1298	L. SALES	0	93
AV.	INT.	48.9423	L. SALES	0	94
AV.	INT.	54.0433	L. SALES	9	95
AV.	INT.	55.1106	L. SALES	57	96
AV.	INT.	54.5769	L. SALES	78	97
AV.	INT.	56.2163	L. SALES	21	98
AV.	INT.	54.5240	L. SALES	39	99
AV.	INT.	53.3269	L. SALES	61	100

RANDOMIZED 2655 DEDICATED 3917

AV.DIFF. 65.9183 MAX.DIFF. 252.0000
TOT. LOST SALES 1045 AV. AGG. INV. 24.8140

AV. INT.	39.1635	L. SALES	0	1
AV. INT.	41.8173	L. SALES	0	2
AV. INT.	38.4567	L. SALES	8	3
AV. INT.	39.2981	L. SALES	22	4
AV. INT.	38.3365	L. SALES	14	5
AV. INT.	38.3125	L. SALES	0	6
AV. INT.	38.2356	L. SALES	4	7
AV. INT.	38.3894	L. SALES	8	8
AV. INT.	38.4375	L. SALES	10	9
AV. INT.	31.5048	L. SALES	4	10
AV. INT.	32.3125	L. SALES	4	11
AV. INT.	33.2500	L. SALES	0	12
AV. INT.	31.8221	L. SALES	0	13
AV. INT.	32.5865	L. SALES	7	14
AV. INT.	31.4856	L. SALES	0	15
AV. INT.	33.3125	L. SALES	0	16
AV. INT.	33.0817	L. SALES	7	17
AV. INT.	33.6971	L. SALES	0	18
AV. INT.	32.4231	L. SALES	1	19
AV. INT.	32.3750	L. SALES	0	20
AV. INT.	32.7115	L. SALES	1	21
AV. INT.	32.2356	L. SALES	0	22
AV. INT.	32.7981	L. SALES	0	23
AV. INT.	31.5817	L. SALES	1	24
AV. INT.	17.9712	L. SALES	0	25
AV. INT.	18.1442	L. SALES	0	26
AV. INT.	18.7692	L. SALES	0	27
AV. INT.	18.9183	L. SALES	0	28
AV. INT.	18.0481	L. SALES	0	29
AV. INT.	18.6394	L. SALES	0	30
AV. INT.	18.4327	L. SALES	0	31
AV. INT.	18.4087	L. SALES	0	32
AV. INT.	18.0721	L. SALES	1	33
AV. INT.	18.1250	L. SALES	0	34
AV. INT.	18.5337	L. SALES	0	35
AV. INT.	18.5913	L. SALES	0	36
AV. INT.	17.7981	L. SALES	0	37
AV. INT.	17.8654	L. SALES	0	38
AV. INT.	18.5577	L. SALES	0	39
AV. INT.	17.9712	L. SALES	0	40
AV. INT.	18.1010	L. SALES	0	41
AV. INT.	17.6298	L. SALES	0	42
AV. INT.	18.6779	L. SALES	0	43
AV. INT.	18.0240	L. SALES	0	44
AV. INT.	18.3942	L. SALES	0	45
AV. INT.	17.4663	L. SALES	0	46
AV. INT.	18.5144	L. SALES	6	47
AV. INT.	17.9904	L. SALES	0	48
AV. INT.	18.1106	L. SALES	0	49
AV. INT.	18.9423	L. SALES	0	50
AV. INT.	18.8221	L. SALES	0	51
AV. INT.	18.6106	L. SALES	0	52
AV. INT.	18.5096	L. SALES	0	53
AV. INT.	19.2212	L. SALES	0	54
AV. INT.	18.4327	L. SALES	0	55
AV. INT.	18.8365	L. SALES	0	56
AV. INT.	17.8462	L. SALES	0	57
AV. INT.	18.9471	L. SALES	0	58
AV. INT.	18.3173	L. SALES	0	59
AV. INT.	17.9712	L. SALES	0	60
AV. INT.	17.2452	L. SALES	0	61
AV. INT.	17.7500	L. SALES	0	62
AV. INT.	17.8654	L. SALES	0	63
AV. INT.	18.5000	L. SALES	0	64
AV. INT.	18.1779	L. SALES	0	65
AV. INT.	18.5385	L. SALES	0	66
AV. INT.	18.5433	L. SALES	0	67
AV. INT.	18.2067	L. SALES	0	68
AV. INT.	18.3942	L. SALES	0	69
AV. INT.	18.2019	L. SALES	0	70
AV. INT.	18.0625	L. SALES	0	71
AV. INT.	18.3654	L. SALES	0	72
AV. INT.	18.1490	L. SALES	0	73
AV. INT.	19.0240	L. SALES	0	74
AV. INT.	18.4808	L. SALES	0	75
AV. INT.	18.1635	L. SALES	0	76
AV. INT.	17.9760	L. SALES	0	77
AV. INT.	19.2019	L. SALES	0	78
AV. INT.	18.6923	L. SALES	0	79
AV. INT.	18.7067	L. SALES	0	80
AV. INT.	17.6298	L. SALES	0	81
AV. INT.	17.8029	L. SALES	0	82
AV. INT.	18.8173	L. SALES	0	83
AV. INT.	18.6538	L. SALES	0	84
AV. INT.	49.9808	L. SALES	5	85
AV. INT.	49.7067	L. SALES	7	86
AV. INT.	49.2548	L. SALES	0	87
AV. INT.	49.0192	L. SALES	0	88
AV. INT.	49.6106	L. SALES	2	89
AV. INT.	50.0817	L. SALES	0	90
AV. INT.	48.5913	L. SALES	0	91
AV. INT.	47.8269	L. SALES	3	92
AV. INT.	48.8317	L. SALES	3	93
AV. INT.	48.4712	L. SALES	0	94
AV. INT.	59.7740	L. SALES	4	95
AV. INT.	60.2212	L. SALES	15	96
AV. INT.	59.2115	L. SALES	15	97
AV. INT.	61.6635	L. SALES	5	98
AV. INT.	59.5385	L. SALES	13	99
AV. INT.	58.0529	L. SALES	6	100

RANDOMIZED 2967 DEDICATED 4210

AV.DIFF. 61.0144 MAX.DIFF. 212.0000

TOT. LOST SALES 176 AV. AGG. INV. 27.8682

AV. INT.	35.2933	L. SALES	9	1
AV. INT.	37.8894	L. SALES	5	2
AV. INT.	34.6442	L. SALES	21	3
AV. INT.	35.4856	L. SALES	35	4
AV. INT.	34.6250	L. SALES	34	5
AV. INT.	34.4856	L. SALES	12	6
AV. INT.	34.3990	L. SALES	13	7
AV. INT.	34.6058	L. SALES	23	8
AV. INT.	34.5962	L. SALES	21	9
AV. INT.	28.3173	L. SALES	14	10
AV. INT.	28.8269	L. SALES	11	11
AV. INT.	29.8702	L. SALES	0	12
AV. INT.	28.3317	L. SALES	2	13
AV. INT.	29.2788	L. SALES	15	14
AV. INT.	27.9087	L. SALES	1	15
AV. INT.	29.8029	L. SALES	3	16
AV. INT.	29.7452	L. SALES	12	17
AV. INT.	30.4808	L. SALES	3	18
AV. INT.	29.1394	L. SALES	5	19
AV. INT.	28.9087	L. SALES	4	20
AV. INT.	29.2067	L. SALES	7	21
AV. INT.	28.8846	L. SALES	5	22
AV. INT.	29.4567	L. SALES	7	23
AV. INT.	28.2452	L. SALES	2	24
AV. INT.	17.9712	L. SALES	0	25
AV. INT.	18.1442	L. SALES	0	26
AV. INT.	18.7692	L. SALES	0	27
AV. INT.	18.9183	L. SALES	0	28
AV. INT.	18.0481	L. SALES	0	29
AV. INT.	18.6394	L. SALES	0	30
AV. INT.	18.4327	L. SALES	0	31
AV. INT.	18.4087	L. SALES	0	32
AV. INT.	18.0721	L. SALES	1	33
AV. INT.	18.1250	L. SALES	0	34
AV. INT.	18.5337	L. SALES	0	35
AV. INT.	18.5913	L. SALES	0	36
AV. INT.	17.7981	L. SALES	0	37
AV. INT.	17.8654	L. SALES	0	38
AV. INT.	18.5577	L. SALES	0	39
AV. INT.	17.9712	L. SALES	0	40
AV. INT.	18.1010	L. SALES	0	41
AV. INT.	17.6298	L. SALES	0	42
AV. INT.	18.6779	L. SALES	0	43
AV. INT.	18.0240	L. SALES	0	44
AV. INT.	18.3942	L. SALES	0	45
AV. INT.	17.4663	L. SALES	0	46
AV. INT.	18.5144	L. SALES	6	47
AV. INT.	17.9904	L. SALES	0	48
AV. INT.	18.1106	L. SALES	0	49
AV. INT.	18.9423	L. SALES	0	50
AV. INT.	18.8221	L. SALES	0	51
AV. INT.	18.6106	L. SALES	0	52
AV. INT.	18.5096	L. SALES	0	53
AV. INT.	19.2212	L. SALES	0	54
AV. INT.	18.4327	L. SALES	0	55
AV. INT.	18.8365	L. SALES	0	56
AV. INT.	17.8462	L. SALES	0	57
AV. INT.	18.9471	L. SALES	0	58
AV. INT.	18.3173	L. SALES	0	59
AV. INT.	17.9712	L. SALES	0	60
AV. INT.	17.2452	L. SALES	0	61
AV. INT.	17.7500	L. SALES	0	62
AV. INT.	17.8654	L. SALES	0	63
AV. INT.	18.5000	L. SALES	0	64
AV. INT.	18.1779	L. SALES	0	65
AV. INT.	18.5385	L. SALES	0	66
AV. INT.	18.5433	L. SALES	0	67
AV. INT.	18.2067	L. SALES	0	68
AV. INT.	18.3942	L. SALES	0	69
AV. INT.	18.2019	L. SALES	0	70
AV. INT.	18.0625	L. SALES	0	71
AV. INT.	18.3654	L. SALES	0	72
AV. INT.	18.1490	L. SALES	0	73
AV. INT.	19.0240	L. SALES	0	74
AV. INT.	18.4808	L. SALES	0	75
AV. INT.	18.1635	L. SALES	0	76
AV. INT.	17.9760	L. SALES	0	77
AV. INT.	19.2019	L. SALES	0	78
AV. INT.	18.6923	L. SALES	0	79
AV. INT.	18.7067	L. SALES	0	80
AV. INT.	17.6298	L. SALES	0	81
AV. INT.	17.8029	L. SALES	0	82
AV. INT.	18.8173	L. SALES	0	83
AV. INT.	18.6538	L. SALES	0	84
AV. INT.	49.9808	L. SALES	5	85
AV. INT.	49.7067	L. SALES	7	86
AV. INT.	49.2548	L. SALES	0	87
AV. INT.	49.0192	L. SALES	0	88
AV. INT.	49.6106	L. SALES	2	89
AV. INT.	50.0817	L. SALES	0	90
AV. INT.	48.5913	L. SALES	0	91
AV. INT.	47.8269	L. SALES	3	92
AV. INT.	48.8317	L. SALES	3	93
AV. INT.	48.4712	L. SALES	0	94
AV. INT.	56.7019	L. SALES	7	95
AV. INT.	57.2596	L. SALES	43	96
AV. INT.	56.2308	L. SALES	50	97
AV. INT.	58.5096	L. SALES	11	98
AV. INT.	56.4904	L. SALES	26	99
AV. INT.	55.2452	L. SALES	27	100

RANDOMIZED 2876 DEDICATED 4104

AV.DIFF. 62.7260 MAX.DIFF. 229.0000
TOT. LOST SALES 455 AV. AGG. INV. 26.8360

AV. INT.	39.0288	L. SALES	23	1
AV. INT.	38.6923	L. SALES	34	2
AV. INT.	38.4471	L. SALES	16	3
AV. INT.	38.4423	L. SALES	19	4
AV. INT.	38.6394	L. SALES	29	5
AV. INT.	38.2596	L. SALES	22	6
AV. INT.	38.8077	L. SALES	15	7
AV. INT.	38.8365	L. SALES	19	8
AV. INT.	38.8702	L. SALES	20	9
AV. INT.	29.3029	L. SALES	0	10
AV. INT.	29.1442	L. SALES	3	11
AV. INT.	29.6010	L. SALES	0	12
AV. INT.	28.8510	L. SALES	8	13
AV. INT.	29.0817	L. SALES	14	14
AV. INT.	28.6394	L. SALES	1	15
AV. INT.	29.4663	L. SALES	2	16
AV. INT.	29.4952	L. SALES	8	17
AV. INT.	29.6971	L. SALES	4	18
AV. INT.	29.2788	L. SALES	4	19
AV. INT.	29.1779	L. SALES	0	20
AV. INT.	29.7163	L. SALES	6	21
AV. INT.	29.4519	L. SALES	8	22
AV. INT.	29.1971	L. SALES	10	23
AV. INT.	29.2885	L. SALES	3	24
AV. INT.	9.4135	L. SALES	28	25
AV. INT.	9.8029	L. SALES	43	26
AV. INT.	9.7837	L. SALES	41	27
AV. INT.	9.7452	L. SALES	42	28
AV. INT.	9.7212	L. SALES	41	29
AV. INT.	9.7500	L. SALES	45	30
AV. INT.	9.8221	L. SALES	33	31
AV. INT.	9.6875	L. SALES	45	32
AV. INT.	9.5577	L. SALES	52	33
AV. INT.	9.7067	L. SALES	47	34
AV. INT.	9.4904	L. SALES	51	35
AV. INT.	9.7067	L. SALES	36	36
AV. INT.	9.5625	L. SALES	78	37
AV. INT.	9.5625	L. SALES	64	38
AV. INT.	9.6683	L. SALES	47	39
AV. INT.	9.5769	L. SALES	49	40
AV. INT.	9.7212	L. SALES	42	41
AV. INT.	9.6394	L. SALES	49	42
AV. INT.	9.8029	L. SALES	41	43
AV. INT.	9.6202	L. SALES	36	44
AV. INT.	9.6298	L. SALES	49	45
AV. INT.	9.5000	L. SALES	50	46
AV. INT.	9.5721	L. SALES	43	47
AV. INT.	9.5673	L. SALES	50	48
AV. INT.	9.5577	L. SALES	45	49
AV. INT.	9.8798	L. SALES	40	50
AV. INT.	9.6394	L. SALES	39	51
AV. INT.	9.6587	L. SALES	51	52
AV. INT.	9.6250	L. SALES	47	53
AV. INT.	9.8029	L. SALES	35	54
AV. INT.	9.6058	L. SALES	44	55
AV. INT.	9.8365	L. SALES	35	56
AV. INT.	9.5385	L. SALES	45	57
AV. INT.	9.8365	L. SALES	49	58
AV. INT.	9.6058	L. SALES	65	59
AV. INT.	9.5048	L. SALES	44	60
AV. INT.	9.5048	L. SALES	58	61
AV. INT.	9.6202	L. SALES	57	62
AV. INT.	9.5144	L. SALES	61	63
AV. INT.	9.6875	L. SALES	35	64
AV. INT.	9.8846	L. SALES	39	65
AV. INT.	9.6971	L. SALES	37	66
AV. INT.	9.6010	L. SALES	33	67
AV. INT.	9.5529	L. SALES	58	68
AV. INT.	9.9038	L. SALES	45	69
AV. INT.	9.7308	L. SALES	48	70
AV. INT.	9.6779	L. SALES	61	71
AV. INT.	9.7308	L. SALES	40	72
AV. INT.	9.6731	L. SALES	46	73
AV. INT.	9.7404	L. SALES	28	74
AV. INT.	9.6587	L. SALES	52	75
AV. INT.	9.7212	L. SALES	58	76
AV. INT.	9.6394	L. SALES	47	77
AV. INT.	9.8077	L. SALES	25	78
AV. INT.	9.7115	L. SALES	35	79
AV. INT.	9.6971	L. SALES	25	80
AV. INT.	9.4663	L. SALES	71	81
AV. INT.	9.5433	L. SALES	49	82
AV. INT.	9.6779	L. SALES	50	83
AV. INT.	9.8125	L. SALES	35	84
AV. INT.	46.3413	L. SALES	0	85
AV. INT.	46.3173	L. SALES	0	86
AV. INT.	46.3413	L. SALES	0	87
AV. INT.	46.2885	L. SALES	0	88
AV. INT.	46.3798	L. SALES	0	89
AV. INT.	46.6683	L. SALES	0	90
AV. INT.	46.2115	L. SALES	0	91
AV. INT.	45.7885	L. SALES	0	92
AV. INT.	46.2260	L. SALES	0	93
AV. INT.	46.1442	L. SALES	0	94
AV. INT.	61.8846	L. SALES	0	95
AV. INT.	61.9471	L. SALES	0	96
AV. INT.	61.6346	L. SALES	0	97
AV. INT.	62.7115	L. SALES	0	98
AV. INT.	62.3702	L. SALES	0	99
AV. INT.	61.5192	L. SALES	7	100

RANDOMIZED 2324 DEDICATED 3187

AV.DIFF. 126.1971 MAX.DIFF. 411.0000
TOT. LOST SALES 3009 AV. AGG. INV. 22.0214

AV. INT.	54.5192	L. SALES	0	1
AV. INT.	55.3413	L. SALES	0	2
AV. INT.	54.4519	L. SALES	0	3
AV. INT.	54.7885	L. SALES	0	4
AV. INT.	54.4279	L. SALES	0	5
AV. INT.	54.5865	L. SALES	0	6
AV. INT.	54.4423	L. SALES	0	7
AV. INT.	54.4519	L. SALES	0	8
AV. INT.	54.3990	L. SALES	0	9
AV. INT.	28.7115	L. SALES	3	10
AV. INT.	28.6779	L. SALES	5	11
AV. INT.	29.0721	L. SALES	4	12
AV. INT.	28.3942	L. SALES	12	13
AV. INT.	28.7981	L. SALES	12	14
AV. INT.	28.0433	L. SALES	1	15
AV. INT.	28.8990	L. SALES	4	16
AV. INT.	29.0769	L. SALES	8	17
AV. INT.	29.3846	L. SALES	6	18
AV. INT.	28.5673	L. SALES	4	19
AV. INT.	28.7933	L. SALES	1	20
AV. INT.	28.9808	L. SALES	6	21
AV. INT.	28.8462	L. SALES	15	22
AV. INT.	28.7019	L. SALES	11	23
AV. INT.	28.8798	L. SALES	3	24
AV. INT.	11.9519	L. SALES	7	25
AV. INT.	11.8365	L. SALES	20	26
AV. INT.	12.2260	L. SALES	8	27
AV. INT.	12.1106	L. SALES	12	28
AV. INT.	11.7644	L. SALES	14	29
AV. INT.	11.9856	L. SALES	20	30
AV. INT.	11.9519	L. SALES	14	31
AV. INT.	12.1635	L. SALES	20	32
AV. INT.	11.8750	L. SALES	25	33
AV. INT.	11.8990	L. SALES	15	34
AV. INT.	11.8221	L. SALES	16	35
AV. INT.	12.0096	L. SALES	18	36
AV. INT.	11.8173	L. SALES	28	37
AV. INT.	11.8894	L. SALES	22	38
AV. INT.	11.9471	L. SALES	11	39
AV. INT.	11.7163	L. SALES	20	40
AV. INT.	12.0433	L. SALES	7	41
AV. INT.	11.8462	L. SALES	17	42
AV. INT.	12.0192	L. SALES	9	43
AV. INT.	11.8798	L. SALES	12	44
AV. INT.	12.0385	L. SALES	17	45
AV. INT.	11.6635	L. SALES	17	46
AV. INT.	11.7933	L. SALES	10	47
AV. INT.	11.6875	L. SALES	24	48
AV. INT.	11.8269	L. SALES	8	49
AV. INT.	12.3654	L. SALES	14	50
AV. INT.	12.1058	L. SALES	5	51
AV. INT.	11.9567	L. SALES	7	52
AV. INT.	12.0048	L. SALES	11	53
AV. INT.	11.8654	L. SALES	7	54
AV. INT.	12.1779	L. SALES	6	55
AV. INT.	12.2356	L. SALES	6	56
AV. INT.	11.7356	L. SALES	16	57
AV. INT.	12.1587	L. SALES	4	58
AV. INT.	12.0529	L. SALES	25	59
AV. INT.	11.8990	L. SALES	15	60
AV. INT.	11.8750	L. SALES	24	61
AV. INT.	11.8510	L. SALES	14	62
AV. INT.	11.9375	L. SALES	28	63
AV. INT.	11.8798	L. SALES	9	64
AV. INT.	12.0192	L. SALES	14	65
AV. INT.	11.8173	L. SALES	11	66
AV. INT.	11.9471	L. SALES	8	67
AV. INT.	12.0625	L. SALES	15	68
AV. INT.	12.0577	L. SALES	17	69
AV. INT.	12.0048	L. SALES	14	70
AV. INT.	11.7692	L. SALES	27	71
AV. INT.	11.8894	L. SALES	16	72
AV. INT.	11.8750	L. SALES	13	73
AV. INT.	12.1683	L. SALES	7	74
AV. INT.	11.9952	L. SALES	19	75
AV. INT.	11.8462	L. SALES	14	76
AV. INT.	11.8077	L. SALES	23	77
AV. INT.	12.1587	L. SALES	8	78
AV. INT.	12.0529	L. SALES	11	79
AV. INT.	12.0962	L. SALES	16	80
AV. INT.	11.8125	L. SALES	22	81
AV. INT.	11.7067	L. SALES	11	82
AV. INT.	12.0240	L. SALES	15	83
AV. INT.	12.1010	L. SALES	13	84
AV. INT.	54.7837	L. SALES	0	85
AV. INT.	54.9423	L. SALES	0	86
AV. INT.	54.6779	L. SALES	0	87
AV. INT.	54.5817	L. SALES	0	88
AV. INT.	54.8990	L. SALES	0	89
AV. INT.	54.9663	L. SALES	0	90
AV. INT.	54.4663	L. SALES	0	91
AV. INT.	54.3125	L. SALES	0	92
AV. INT.	54.5673	L. SALES	0	93
AV. INT.	54.5721	L. SALES	0	94
AV. INT.	71.6106	L. SALES	0	95
AV. INT.	71.7644	L. SALES	0	96
AV. INT.	70.9760	L. SALES	0	97
AV. INT.	72.2740	L. SALES	0	98
AV. INT.	71.4904	L. SALES	0	99
AV. INT.	70.6442	L. SALES	0	100

RANDOMIZED 2776 DEDICATED 3581

AV.DIFF. 117.7308 MAX.DIFF. 443.0000
TOT. LOST SALES 971 AV. AGG. INV. 26.1584

AV. INT.	60.0577	L. SALES	0	1
AV. INT.	60.9279	L. SALES	0	2
AV. INT.	59.7115	L. SALES	0	3
AV. INT.	59.9567	L. SALES	0	4
AV. INT.	59.6971	L. SALES	0	5
AV. INT.	59.7452	L. SALES	0	6
AV. INT.	59.7163	L. SALES	0	7
AV. INT.	59.7644	L. SALES	0	8
AV. INT.	59.7644	L. SALES	0	9
AV. INT.	34.7644	L. SALES	0	10
AV. INT.	34.6202	L. SALES	0	11
AV. INT.	35.0000	L. SALES	0	12
AV. INT.	34.7212	L. SALES	0	13
AV. INT.	34.9760	L. SALES	0	14
AV. INT.	34.3750	L. SALES	0	15
AV. INT.	34.7260	L. SALES	0	16
AV. INT.	34.7933	L. SALES	0	17
AV. INT.	35.2644	L. SALES	0	18
AV. INT.	34.6875	L. SALES	0	19
AV. INT.	35.0192	L. SALES	0	20
AV. INT.	34.9471	L. SALES	1	21
AV. INT.	34.9808	L. SALES	0	22
AV. INT.	34.8894	L. SALES	0	23
AV. INT.	34.6298	L. SALES	0	24
AV. INT.	16.7596	L. SALES	0	25
AV. INT.	16.7596	L. SALES	0	26
AV. INT.	16.9471	L. SALES	0	27
AV. INT.	16.9183	L. SALES	0	28
AV. INT.	16.7260	L. SALES	0	29
AV. INT.	16.9760	L. SALES	0	30
AV. INT.	16.9183	L. SALES	0	31
AV. INT.	16.9856	L. SALES	0	32
AV. INT.	16.7740	L. SALES	0	33
AV. INT.	16.8413	L. SALES	0	34
AV. INT.	16.9856	L. SALES	0	35
AV. INT.	17.0240	L. SALES	0	36
AV. INT.	16.8750	L. SALES	0	37
AV. INT.	16.5721	L. SALES	0	38
AV. INT.	16.7933	L. SALES	0	39
AV. INT.	16.4087	L. SALES	0	40
AV. INT.	16.6779	L. SALES	0	41
AV. INT.	16.5625	L. SALES	0	42
AV. INT.	16.9327	L. SALES	0	43
AV. INT.	16.8173	L. SALES	0	44
AV. INT.	16.9327	L. SALES	0	45
AV. INT.	16.7308	L. SALES	0	46
AV. INT.	16.8510	L. SALES	3	47
AV. INT.	16.8462	L. SALES	0	48
AV. INT.	16.5577	L. SALES	0	49
AV. INT.	17.1683	L. SALES	1	50
AV. INT.	17.1779	L. SALES	0	51
AV. INT.	17.1010	L. SALES	2	52
AV. INT.	16.8077	L. SALES	0	53
AV. INT.	17.1394	L. SALES	0	54
AV. INT.	16.8462	L. SALES	0	55
AV. INT.	17.0529	L. SALES	0	56
AV. INT.	16.6731	L. SALES	0	57
AV. INT.	17.0962	L. SALES	0	58
AV. INT.	16.8077	L. SALES	0	59
AV. INT.	16.6298	L. SALES	0	60
AV. INT.	16.4231	L. SALES	6	61
AV. INT.	16.6394	L. SALES	0	62
AV. INT.	16.8510	L. SALES	0	63
AV. INT.	17.0385	L. SALES	0	64
AV. INT.	16.8990	L. SALES	0	65
AV. INT.	17.0096	L. SALES	0	66
AV. INT.	16.9808	L. SALES	0	67
AV. INT.	16.8990	L. SALES	2	68
AV. INT.	16.9712	L. SALES	0	69
AV. INT.	16.8462	L. SALES	0	70
AV. INT.	16.6731	L. SALES	0	71
AV. INT.	16.7788	L. SALES	0	72
AV. INT.	16.8942	L. SALES	0	73
AV. INT.	17.1538	L. SALES	1	74
AV. INT.	16.9375	L. SALES	0	75
AV. INT.	16.8077	L. SALES	0	76
AV. INT.	16.8846	L. SALES	0	77
AV. INT.	17.3125	L. SALES	0	78
AV. INT.	16.8558	L. SALES	0	79
AV. INT.	16.7548	L. SALES	0	80
AV. INT.	16.5577	L. SALES	1	81
AV. INT.	16.6154	L. SALES	0	82
AV. INT.	17.1010	L. SALES	0	83
AV. INT.	16.9327	L. SALES	0	84
AV. INT.	61.7837	L. SALES	0	85
AV. INT.	61.9423	L. SALES	0	86
AV. INT.	61.6779	L. SALES	0	87
AV. INT.	61.5817	L. SALES	0	88
AV. INT.	61.8990	L. SALES	0	89
AV. INT.	61.9663	L. SALES	0	90
AV. INT.	61.4663	L. SALES	0	91
AV. INT.	61.3125	L. SALES	0	92
AV. INT.	61.5673	L. SALES	0	93
AV. INT.	61.5721	L. SALES	0	94
AV. INT.	79.4519	L. SALES	0	95
AV. INT.	78.9183	L. SALES	0	96
AV. INT.	78.3750	L. SALES	0	97
AV. INT.	79.4567	L. SALES	0	98
AV. INT.	78.6490	L. SALES	0	99
AV. INT.	77.9615	L. SALES	0	100

RANDOMIZED 3273 DEDICATED 4078

AV.DIFF. 104.4375 MAX.DIFF. 369.0000
TOT. LOST SALES 17 AV. AGG. INV. 31.6281

AV.	INT.	64.0577	L. SALES	0	1
AV.	INT.	64.9712	L. SALES	0	2
AV.	INT.	63.7548	L. SALES	0	3
AV.	INT.	63.9567	L. SALES	0	4
AV.	INT.	63.6971	L. SALES	0	5
AV.	INT.	63.7452	L. SALES	0	6
AV.	INT.	63.7163	L. SALES	0	7
AV.	INT.	63.7644	L. SALES	0	8
AV.	INT.	63.7644	L. SALES	0	9
AV.	INT.	39.0962	L. SALES	0	10
AV.	INT.	39.1490	L. SALES	0	11
AV.	INT.	39.2981	L. SALES	0	12
AV.	INT.	39.3221	L. SALES	0	13
AV.	INT.	39.5962	L. SALES	0	14
AV.	INT.	38.8750	L. SALES	0	15
AV.	INT.	39.4279	L. SALES	0	16
AV.	INT.	39.5865	L. SALES	0	17
AV.	INT.	39.7452	L. SALES	0	18
AV.	INT.	39.4856	L. SALES	0	19
AV.	INT.	39.6587	L. SALES	0	20
AV.	INT.	39.3510	L. SALES	0	21
AV.	INT.	39.5721	L. SALES	0	22
AV.	INT.	39.5481	L. SALES	0	23
AV.	INT.	39.3846	L. SALES	0	24
AV.	INT.	19.1731	L. SALES	0	25
AV.	INT.	19.3750	L. SALES	0	26
AV.	INT.	19.3510	L. SALES	0	27
AV.	INT.	19.4904	L. SALES	0	28
AV.	INT.	19.3269	L. SALES	0	29
AV.	INT.	19.4519	L. SALES	0	30
AV.	INT.	19.4327	L. SALES	0	31
AV.	INT.	19.2788	L. SALES	0	32
AV.	INT.	19.2692	L. SALES	0	33
AV.	INT.	19.1442	L. SALES	0	34
AV.	INT.	19.4663	L. SALES	0	35
AV.	INT.	19.3125	L. SALES	0	36
AV.	INT.	19.0529	L. SALES	0	37
AV.	INT.	19.1731	L. SALES	0	38
AV.	INT.	19.3462	L. SALES	0	39
AV.	INT.	19.1058	L. SALES	0	40
AV.	INT.	19.0769	L. SALES	0	41
AV.	INT.	19.2452	L. SALES	0	42
AV.	INT.	19.4087	L. SALES	0	43
AV.	INT.	19.3558	L. SALES	0	44
AV.	INT.	19.3846	L. SALES	0	45
AV.	INT.	19.0817	L. SALES	0	46
AV.	INT.	19.3894	L. SALES	1	47
AV.	INT.	19.3654	L. SALES	0	48
AV.	INT.	19.2596	L. SALES	0	49
AV.	INT.	19.4856	L. SALES	0	50
AV.	INT.	19.5240	L. SALES	0	51
AV.	INT.	19.3462	L. SALES	0	52
AV.	INT.	19.2788	L. SALES	0	53
AV.	INT.	19.6250	L. SALES	0	54
AV.	INT.	19.2260	L. SALES	0	55
AV.	INT.	19.5000	L. SALES	0	56
AV.	INT.	19.1346	L. SALES	0	57
AV.	INT.	19.4856	L. SALES	0	58
AV.	INT.	19.3077	L. SALES	0	59
AV.	INT.	19.1346	L. SALES	0	60
AV.	INT.	19.1490	L. SALES	0	61
AV.	INT.	19.2067	L. SALES	0	62
AV.	INT.	19.1346	L. SALES	0	63
AV.	INT.	19.4808	L. SALES	0	64
AV.	INT.	19.2212	L. SALES	0	65
AV.	INT.	19.4375	L. SALES	0	66
AV.	INT.	19.3894	L. SALES	0	67
AV.	INT.	19.3029	L. SALES	0	68
AV.	INT.	19.4375	L. SALES	0	69
AV.	INT.	19.3317	L. SALES	0	70
AV.	INT.	19.3269	L. SALES	0	71
AV.	INT.	19.3702	L. SALES	0	72
AV.	INT.	19.2596	L. SALES	0	73
AV.	INT.	19.4567	L. SALES	0	74
AV.	INT.	19.3702	L. SALES	0	75
AV.	INT.	19.2500	L. SALES	0	76
AV.	INT.	19.2452	L. SALES	0	77
AV.	INT.	19.5962	L. SALES	0	78
AV.	INT.	19.4567	L. SALES	0	79
AV.	INT.	19.5577	L. SALES	0	80
AV.	INT.	19.1490	L. SALES	0	81
AV.	INT.	19.2115	L. SALES	0	82
AV.	INT.	19.5865	L. SALES	0	83
AV.	INT.	19.5481	L. SALES	0	84
AV.	INT.	65.7837	L. SALES	0	85
AV.	INT.	65.9423	L. SALES	0	86
AV.	INT.	65.6779	L. SALES	0	87
AV.	INT.	65.5817	L. SALES	0	88
AV.	INT.	65.8990	L. SALES	0	89
AV.	INT.	65.9663	L. SALES	0	90
AV.	INT.	65.4663	L. SALES	0	91
AV.	INT.	65.3125	L. SALES	0	92
AV.	INT.	65.5673	L. SALES	0	93
AV.	INT.	65.5721	L. SALES	0	94
AV.	INT.	83.4519	L. SALES	0	95
AV.	INT.	82.9183	L. SALES	0	96
AV.	INT.	82.3750	L. SALES	0	97
AV.	INT.	83.4567	L. SALES	0	98
AV.	INT.	82.6490	L. SALES	0	99
AV.	INT.	81.9615	L. SALES	0	100

RANDOMIZED 3603 DEDICATED 4363

AV.DIFF. 87.6923 MAX.DIFF. 331.0000
TOT. LOST SALES 1 AV. AGG. INV. 34.7995

AV. INT.	26.6442	L. SALES	172	1
AV. INT.	28.0144	L. SALES	83	2
AV. INT.	25.7548	L. SALES	170	3
AV. INT.	25.9904	L. SALES	142	4
AV. INT.	26.0625	L. SALES	203	5
AV. INT.	25.7596	L. SALES	171	6
AV. INT.	25.7452	L. SALES	178	7
AV. INT.	25.8365	L. SALES	176	8
AV. INT.	25.5721	L. SALES	158	9
AV. INT.	18.2740	L. SALES	250	10
AV. INT.	18.0433	L. SALES	223	11
AV. INT.	18.7404	L. SALES	191	12
AV. INT.	18.4567	L. SALES	265	13
AV. INT.	19.1587	L. SALES	277	14
AV. INT.	17.8558	L. SALES	275	15
AV. INT.	18.7692	L. SALES	196	16
AV. INT.	18.5673	L. SALES	203	17
AV. INT.	18.9760	L. SALES	164	18
AV. INT.	18.5000	L. SALES	218	19
AV. INT.	18.8173	L. SALES	248	20
AV. INT.	18.6010	L. SALES	235	21
AV. INT.	18.8221	L. SALES	231	22
AV. INT.	18.7933	L. SALES	207	23
AV. INT.	18.4327	L. SALES	263	24
AV. INT.	11.2596	L. SALES	18	25
AV. INT.	11.7308	L. SALES	31	26
AV. INT.	11.9760	L. SALES	15	27
AV. INT.	12.1106	L. SALES	10	28
AV. INT.	11.4087	L. SALES	17	29
AV. INT.	12.1442	L. SALES	31	30
AV. INT.	11.7788	L. SALES	18	31
AV. INT.	11.8702	L. SALES	33	32
AV. INT.	11.5481	L. SALES	32	33
AV. INT.	11.4760	L. SALES	21	34
AV. INT.	11.7740	L. SALES	18	35
AV. INT.	11.7548	L. SALES	24	36
AV. INT.	11.3365	L. SALES	38	37
AV. INT.	11.2404	L. SALES	22	38
AV. INT.	11.7404	L. SALES	12	39
AV. INT.	11.5192	L. SALES	35	40
AV. INT.	11.3942	L. SALES	21	41
AV. INT.	11.3029	L. SALES	31	42
AV. INT.	11.8894	L. SALES	16	43
AV. INT.	11.4327	L. SALES	28	44
AV. INT.	11.7356	L. SALES	23	45
AV. INT.	11.1442	L. SALES	39	46
AV. INT.	11.8029	L. SALES	29	47
AV. INT.	11.4615	L. SALES	25	48
AV. INT.	11.3654	L. SALES	27	49
AV. INT.	12.1731	L. SALES	11	50
AV. INT.	12.0048	L. SALES	8	51
AV. INT.	11.7837	L. SALES	9	52
AV. INT.	11.8413	L. SALES	23	53
AV. INT.	12.3894	L. SALES	9	54
AV. INT.	11.7019	L. SALES	21	55
AV. INT.	12.0673	L. SALES	11	56
AV. INT.	11.1058	L. SALES	23	57
AV. INT.	12.3413	L. SALES	28	58
AV. INT.	11.5913	L. SALES	29	59
AV. INT.	11.1875	L. SALES	20	60
AV. INT.	10.9712	L. SALES	40	61
AV. INT.	11.2644	L. SALES	34	62
AV. INT.	11.2596	L. SALES	22	63
AV. INT.	11.7308	L. SALES	13	64
AV. INT.	11.8221	L. SALES	36	65
AV. INT.	11.8029	L. SALES	10	66
AV. INT.	11.6635	L. SALES	15	67
AV. INT.	11.7115	L. SALES	37	68
AV. INT.	11.9760	L. SALES	30	69
AV. INT.	11.5913	L. SALES	23	70
AV. INT.	11.5433	L. SALES	25	71
AV. INT.	11.8029	L. SALES	31	72
AV. INT.	11.4808	L. SALES	28	73
AV. INT.	12.2212	L. SALES	14	74
AV. INT.	11.8317	L. SALES	29	75
AV. INT.	11.5721	L. SALES	30	76
AV. INT.	11.3894	L. SALES	31	77
AV. INT.	12.4856	L. SALES	14	78
AV. INT.	11.9760	L. SALES	16	79
AV. INT.	11.9231	L. SALES	8	80
AV. INT.	11.1346	L. SALES	38	81
AV. INT.	11.3029	L. SALES	25	82
AV. INT.	12.1587	L. SALES	10	83
AV. INT.	11.9663	L. SALES	15	84
AV. INT.	42.8125	L. SALES	32	85
AV. INT.	42.7067	L. SALES	33	86
AV. INT.	41.6202	L. SALES	0	87
AV. INT.	41.3125	L. SALES	0	88
AV. INT.	42.4087	L. SALES	21	89
AV. INT.	42.3798	L. SALES	0	90
AV. INT.	41.1490	L. SALES	24	91
AV. INT.	40.7692	L. SALES	32	92
AV. INT.	41.3606	L. SALES	16	93
AV. INT.	40.9423	L. SALES	0	94
AV. INT.	43.2644	L. SALES	38	95
AV. INT.	45.8558	L. SALES	201	96
AV. INT.	45.3990	L. SALES	230	97
AV. INT.	45.4327	L. SALES	58	98
AV. INT.	44.5288	L. SALES	151	99
AV. INT.	43.8125	L. SALES	212	100

RANDOMIZED 2105 DEDICATED 3417

AV.DIFF. 92.7644 MAX.DIFF. 329.0000

TOT. LOST SALES 7327 AV. AGG. INV. 19.0091

AV. INT.	44.1346	L. SALES	0	1
AV. INT.	45.8798	L. SALES	0	2
AV. INT.	43.5337	L. SALES	0	3
AV. INT.	43.9567	L. SALES	3	4
AV. INT.	43.4135	L. SALES	0	5
AV. INT.	43.5385	L. SALES	0	6
AV. INT.	43.4615	L. SALES	0	7
AV. INT.	43.5337	L. SALES	0	8
AV. INT.	43.5144	L. SALES	0	9
AV. INT.	26.8990	L. SALES	13	10
AV. INT.	27.0817	L. SALES	5	11
AV. INT.	27.6731	L. SALES	15	12
AV. INT.	27.2885	L. SALES	20	13
AV. INT.	27.7885	L. SALES	28	14
AV. INT.	26.5865	L. SALES	24	15
AV. INT.	27.8365	L. SALES	14	16
AV. INT.	27.8413	L. SALES	8	17
AV. INT.	28.2356	L. SALES	6	18
AV. INT.	27.5192	L. SALES	17	19
AV. INT.	27.5577	L. SALES	1	20
AV. INT.	27.4952	L. SALES	16	21
AV. INT.	27.7019	L. SALES	18	22
AV. INT.	27.8173	L. SALES	24	23
AV. INT.	27.4231	L. SALES	28	24
AV. INT.	15.0962	L. SALES	1	25
AV. INT.	15.3269	L. SALES	4	26
AV. INT.	15.5481	L. SALES	1	27
AV. INT.	15.7115	L. SALES	0	28
AV. INT.	15.2788	L. SALES	4	29
AV. INT.	15.6106	L. SALES	5	30
AV. INT.	15.6154	L. SALES	6	31
AV. INT.	15.3413	L. SALES	3	32
AV. INT.	15.2260	L. SALES	5	33
AV. INT.	15.1106	L. SALES	1	34
AV. INT.	15.5481	L. SALES	0	35
AV. INT.	15.3798	L. SALES	2	36
AV. INT.	14.8798	L. SALES	1	37
AV. INT.	15.0000	L. SALES	0	38
AV. INT.	15.4663	L. SALES	0	39
AV. INT.	15.0385	L. SALES	1	40
AV. INT.	15.0625	L. SALES	0	41
AV. INT.	14.9712	L. SALES	1	42
AV. INT.	15.5192	L. SALES	0	43
AV. INT.	15.2596	L. SALES	0	44
AV. INT.	15.4135	L. SALES	1	45
AV. INT.	14.8558	L. SALES	4	46
AV. INT.	15.4856	L. SALES	3	47
AV. INT.	15.2308	L. SALES	1	48
AV. INT.	15.1202	L. SALES	0	49
AV. INT.	15.7644	L. SALES	2	50
AV. INT.	15.6923	L. SALES	0	51
AV. INT.	15.4760	L. SALES	1	52
AV. INT.	15.3702	L. SALES	1	53
AV. INT.	15.9279	L. SALES	0	54
AV. INT.	15.3125	L. SALES	1	55
AV. INT.	15.7067	L. SALES	0	56
AV. INT.	14.9663	L. SALES	0	57
AV. INT.	15.7163	L. SALES	2	58
AV. INT.	15.3317	L. SALES	1	59
AV. INT.	15.0625	L. SALES	2	60
AV. INT.	14.8173	L. SALES	6	61
AV. INT.	14.9904	L. SALES	2	62
AV. INT.	15.0625	L. SALES	2	63
AV. INT.	15.5865	L. SALES	0	64
AV. INT.	15.2115	L. SALES	2	65
AV. INT.	15.5000	L. SALES	0	66
AV. INT.	15.4856	L. SALES	0	67
AV. INT.	15.3029	L. SALES	4	68
AV. INT.	15.4615	L. SALES	0	69
AV. INT.	15.3077	L. SALES	1	70
AV. INT.	15.3317	L. SALES	3	71
AV. INT.	15.4135	L. SALES	0	72
AV. INT.	15.2019	L. SALES	4	73
AV. INT.	15.7163	L. SALES	0	74
AV. INT.	15.4279	L. SALES	0	75
AV. INT.	15.2019	L. SALES	2	76
AV. INT.	15.2500	L. SALES	3	77
AV. INT.	15.9231	L. SALES	0	78
AV. INT.	15.5817	L. SALES	0	79
AV. INT.	15.7163	L. SALES	0	80
AV. INT.	14.9087	L. SALES	0	81
AV. INT.	15.0865	L. SALES	0	82
AV. INT.	15.8125	L. SALES	1	83
AV. INT.	15.6490	L. SALES	0	84
AV. INT.	54.0673	L. SALES	0	85
AV. INT.	54.1250	L. SALES	0	86
AV. INT.	53.6106	L. SALES	0	87
AV. INT.	53.4135	L. SALES	0	88
AV. INT.	53.9808	L. SALES	0	89
AV. INT.	54.1779	L. SALES	0	90
AV. INT.	53.1490	L. SALES	0	91
AV. INT.	52.8029	L. SALES	0	92
AV. INT.	53.3606	L. SALES	0	93
AV. INT.	53.2404	L. SALES	0	94
AV. INT.	63.0962	L. SALES	0	95
AV. INT.	62.9856	L. SALES	0	96
AV. INT.	62.2981	L. SALES	16	97
AV. INT.	64.0048	L. SALES	0	98
AV. INT.	62.4856	L. SALES	0	99
AV. INT.	61.4760	L. SALES	9	100

RANDOMIZED 2816 DEDICATED 3906

AV.DIFF. 90.2500 MAX.DIFF. 359.0000
TOT. LOST SALES 349 AV. AGG. INV. 26.4136

AV.	INT.	46.1346	L. SALES	0	1
AV.	INT.	47.8798	L. SALES	0	2
AV.	INT.	45.5721	L. SALES	0	3
AV.	INT.	45.9760	L. SALES	1	4
AV.	INT.	45.4135	L. SALES	0	5
AV.	INT.	45.5385	L. SALES	0	6
AV.	INT.	45.4615	L. SALES	0	7
AV.	INT.	45.5337	L. SALES	0	8
AV.	INT.	45.5144	L. SALES	0	9
AV.	INT.	30.2019	L. SALES	2	10
AV.	INT.	30.7163	L. SALES	1	11
AV.	INT.	31.3269	L. SALES	2	12
AV.	INT.	30.4856	L. SALES	5	13
AV.	INT.	30.7500	L. SALES	4	14
AV.	INT.	30.3077	L. SALES	5	15
AV.	INT.	31.2500	L. SALES	0	16
AV.	INT.	30.9856	L. SALES	5	17
AV.	INT.	31.6683	L. SALES	0	18
AV.	INT.	30.8462	L. SALES	1	19
AV.	INT.	30.8317	L. SALES	0	20
AV.	INT.	30.8798	L. SALES	6	21
AV.	INT.	30.7163	L. SALES	0	22
AV.	INT.	31.1923	L. SALES	6	23
AV.	INT.	30.6442	L. SALES	3	24
AV.	INT.	16.5721	L. SALES	0	25
AV.	INT.	16.6875	L. SALES	3	26
AV.	INT.	16.9135	L. SALES	0	27
AV.	INT.	17.3269	L. SALES	0	28
AV.	INT.	16.6923	L. SALES	0	29
AV.	INT.	16.9471	L. SALES	2	30
AV.	INT.	16.8510	L. SALES	0	31
AV.	INT.	16.8798	L. SALES	0	32
AV.	INT.	16.6971	L. SALES	0	33
AV.	INT.	16.6010	L. SALES	0	34
AV.	INT.	16.9615	L. SALES	0	35
AV.	INT.	16.8894	L. SALES	1	36
AV.	INT.	16.5529	L. SALES	0	37
AV.	INT.	16.4231	L. SALES	0	38
AV.	INT.	17.0337	L. SALES	0	39
AV.	INT.	16.6346	L. SALES	0	40
AV.	INT.	16.8317	L. SALES	0	41
AV.	INT.	16.4519	L. SALES	0	42
AV.	INT.	17.1106	L. SALES	0	43
AV.	INT.	16.6923	L. SALES	0	44
AV.	INT.	16.7885	L. SALES	0	45
AV.	INT.	16.2885	L. SALES	1	46
AV.	INT.	16.8654	L. SALES	0	47
AV.	INT.	16.6442	L. SALES	0	48
AV.	INT.	16.6394	L. SALES	0	49
AV.	INT.	17.2404	L. SALES	0	50
AV.	INT.	17.0240	L. SALES	0	51
AV.	INT.	17.0673	L. SALES	0	52
AV.	INT.	16.9279	L. SALES	2	53
AV.	INT.	17.4135	L. SALES	0	54
AV.	INT.	16.9327	L. SALES	1	55
AV.	INT.	17.2981	L. SALES	0	56
AV.	INT.	16.3462	L. SALES	0	57
AV.	INT.	17.1827	L. SALES	0	58
AV.	INT.	16.9327	L. SALES	1	59
AV.	INT.	16.6779	L. SALES	0	60
AV.	INT.	16.2067	L. SALES	3	61
AV.	INT.	16.3654	L. SALES	0	62
AV.	INT.	16.6106	L. SALES	1	63
AV.	INT.	16.8990	L. SALES	0	64
AV.	INT.	16.6490	L. SALES	1	65
AV.	INT.	16.9135	L. SALES	0	66
AV.	INT.	16.9038	L. SALES	0	67
AV.	INT.	16.8413	L. SALES	3	68
AV.	INT.	16.8510	L. SALES	0	69
AV.	INT.	16.7548	L. SALES	0	70
AV.	INT.	16.4856	L. SALES	0	71
AV.	INT.	16.7837	L. SALES	0	72
AV.	INT.	16.8558	L. SALES	0	73
AV.	INT.	17.1923	L. SALES	0	74
AV.	INT.	16.9183	L. SALES	0	75
AV.	INT.	16.7212	L. SALES	0	76
AV.	INT.	16.6154	L. SALES	0	77
AV.	INT.	17.4038	L. SALES	0	78
AV.	INT.	16.9808	L. SALES	3	79
AV.	INT.	17.1250	L. SALES	0	80
AV.	INT.	16.3077	L. SALES	0	81
AV.	INT.	16.5433	L. SALES	0	82
AV.	INT.	17.0962	L. SALES	0	83
AV.	INT.	17.0288	L. SALES	0	84
AV.	INT.	56.0673	L. SALES	0	85
AV.	INT.	56.1250	L. SALES	0	86
AV.	INT.	55.6106	L. SALES	0	87
AV.	INT.	55.4135	L. SALES	0	88
AV.	INT.	55.9808	L. SALES	0	89
AV.	INT.	56.1779	L. SALES	0	90
AV.	INT.	55.1490	L. SALES	0	91
AV.	INT.	54.8029	L. SALES	0	92
AV.	INT.	55.3606	L. SALES	0	93
AV.	INT.	55.2404	L. SALES	0	94
AV.	INT.	65.2019	L. SALES	0	95
AV.	INT.	65.4183	L. SALES	0	96
AV.	INT.	64.4038	L. SALES	3	97
AV.	INT.	66.4808	L. SALES	0	98
AV.	INT.	64.8894	L. SALES	0	99
AV.	INT.	63.6058	L. SALES	7	100

RANDOMIZED 3004 DEDICATED 4047

AV.DIFF. 76.2115 MAX.DIFF. 239.0000
TOT. LOST SALES 73 AV. AGG. INV. 28.3083

AV. INT.	48.1346	L. SALES	0	1
AV. INT.	49.8798	L. SALES	0	2
AV. INT.	47.5721	L. SALES	0	3
AV. INT.	47.9663	L. SALES	0	4
AV. INT.	47.4135	L. SALES	0	5
AV. INT.	47.5385	L. SALES	0	6
AV. INT.	47.4615	L. SALES	0	7
AV. INT.	47.5337	L. SALES	0	8
AV. INT.	47.5144	L. SALES	0	9
AV. INT.	32.8750	L. SALES	2	10
AV. INT.	33.4087	L. SALES	0	11
AV. INT.	33.9663	L. SALES	0	12
AV. INT.	33.1346	L. SALES	1	13
AV. INT.	33.5337	L. SALES	0	14
AV. INT.	33.0048	L. SALES	0	15
AV. INT.	34.1058	L. SALES	0	16
AV. INT.	33.8846	L. SALES	5	17
AV. INT.	34.2212	L. SALES	0	18
AV. INT.	33.4183	L. SALES	0	19
AV. INT.	33.4279	L. SALES	0	20
AV. INT.	33.6538	L. SALES	3	21
AV. INT.	33.2740	L. SALES	3	22
AV. INT.	33.6923	L. SALES	0	23
AV. INT.	32.8125	L. SALES	0	24
AV. INT.	17.9567	L. SALES	0	25
AV. INT.	18.1875	L. SALES	0	26
AV. INT.	18.5385	L. SALES	0	27
AV. INT.	18.7788	L. SALES	0	28
AV. INT.	18.1538	L. SALES	0	29
AV. INT.	18.4760	L. SALES	1	30
AV. INT.	18.2692	L. SALES	0	31
AV. INT.	18.4038	L. SALES	0	32
AV. INT.	18.0913	L. SALES	0	33
AV. INT.	18.1971	L. SALES	0	34
AV. INT.	18.4471	L. SALES	0	35
AV. INT.	18.2212	L. SALES	0	36
AV. INT.	17.8654	L. SALES	0	37
AV. INT.	18.1298	L. SALES	0	38
AV. INT.	18.5577	L. SALES	0	39
AV. INT.	18.2788	L. SALES	0	40
AV. INT.	18.2260	L. SALES	0	41
AV. INT.	17.7356	L. SALES	0	42
AV. INT.	18.5192	L. SALES	0	43
AV. INT.	18.1442	L. SALES	0	44
AV. INT.	18.2163	L. SALES	0	45
AV. INT.	17.7596	L. SALES	0	46
AV. INT.	18.3462	L. SALES	0	47
AV. INT.	18.1202	L. SALES	0	48
AV. INT.	18.0577	L. SALES	0	49
AV. INT.	18.7740	L. SALES	0	50
AV. INT.	18.5385	L. SALES	0	51
AV. INT.	18.4183	L. SALES	0	52
AV. INT.	18.3221	L. SALES	1	53
AV. INT.	18.9038	L. SALES	0	54
AV. INT.	18.2212	L. SALES	0	55
AV. INT.	18.6683	L. SALES	0	56
AV. INT.	17.8125	L. SALES	0	57
AV. INT.	18.8029	L. SALES	0	58
AV. INT.	18.3269	L. SALES	0	59
AV. INT.	18.2452	L. SALES	0	60
AV. INT.	17.6827	L. SALES	2	61
AV. INT.	17.8798	L. SALES	0	62
AV. INT.	18.0048	L. SALES	0	63
AV. INT.	18.5385	L. SALES	0	64
AV. INT.	18.2308	L. SALES	0	65
AV. INT.	18.3269	L. SALES	0	66
AV. INT.	18.5337	L. SALES	0	67
AV. INT.	18.3077	L. SALES	0	68
AV. INT.	18.3942	L. SALES	0	69
AV. INT.	18.2596	L. SALES	0	70
AV. INT.	18.1442	L. SALES	0	71
AV. INT.	18.3365	L. SALES	0	72
AV. INT.	18.1154	L. SALES	0	73
AV. INT.	18.5962	L. SALES	0	74
AV. INT.	18.4327	L. SALES	0	75
AV. INT.	18.1635	L. SALES	0	76
AV. INT.	18.0337	L. SALES	0	77
AV. INT.	18.7644	L. SALES	0	78
AV. INT.	18.3846	L. SALES	0	79
AV. INT.	18.5673	L. SALES	0	80
AV. INT.	17.8413	L. SALES	0	81
AV. INT.	18.0817	L. SALES	0	82
AV. INT.	18.8846	L. SALES	0	83
AV. INT.	18.5337	L. SALES	0	84
AV. INT.	58.0673	L. SALES	0	85
AV. INT.	58.1250	L. SALES	0	86
AV. INT.	57.6106	L. SALES	0	87
AV. INT.	57.4135	L. SALES	0	88
AV. INT.	57.9808	L. SALES	0	89
AV. INT.	58.1779	L. SALES	0	90
AV. INT.	57.1490	L. SALES	0	91
AV. INT.	56.8029	L. SALES	0	92
AV. INT.	57.3606	L. SALES	0	93
AV. INT.	57.2404	L. SALES	0	94
AV. INT.	67.2500	L. SALES	0	95
AV. INT.	67.6587	L. SALES	0	96
AV. INT.	66.7019	L. SALES	0	97
AV. INT.	68.7356	L. SALES	0	98
AV. INT.	67.2404	L. SALES	0	99
AV. INT.	65.8413	L. SALES	0	100

RANDOMIZED 3143 DEDICATED 4190

AV.DIFF. 62.1010 MAX.DIFF. 223.0000
TOT. LOST SALES 18 AV. AGG. INV. 30.1053

APPENDIX 3

Output of all possible regressions based on simulation runs

INPUT DATA

62.726	7.6914	229	52441	26.036	1.427
61.0144	5.6041	212	44944	27.8682	1.4189
65.9183	10.1478	252	53424	24.814	1.4753
62.101	2.6207	223	49729	30.1053	1.3331
76.2115	4.1793	239	57121	28.3083	1.3472
90.25	7.0406	359	128881	26.4136	1.3871
92.7644	19.4226	329	100241	19.0091	1.6233
87.6923	1	331	109561	34.7995	1.2109
104.438	2.5713	369	136161	31.6281	1.246
117.731	9.9023	443	196249	26.1584	1.29
126.197	14.4369	411	168921	22.0214	1.3713

SUMS OF VARIABLES

947.043	84.617	3397	1105673	297.962	15.1301

MEANS OF VARIABLES

86.0948	7.69245	308.818	100516.	27.0874	1.37546

SUMS OF CROSS PRODUCTS

86800.9	7755.64	310478.	1.06917E+8	25407.	1295.8
7755.64	958.775	27607.6	9.41781E+6	2059.84	121.303
310478.	27607.6	1115753	384493957	91350.6	4646.32
1.06917E+8	9.41781E+6	384493957	1.39278E+11	2.95340E+7	1.50227E+6
25407.	2059.84	91350.6	2.95340E+7	8259.8	405.701
1295.8	121.303	4646.32	1.50227E+6	405.701	20.9407

CORRECTED S S MATRIX

5265.39	470.552	18014.2	1.17243E+7	-245.934	-6.81927
470.552	307.862	1476.35	912468.	-232.215	4.91549
18014.2	1476.35	66697.6	4.30420E+7	-665.41	-26.1317
1.17243E+7	912468.	4.30420E+7	2.01402E+10	-415880.	-18540.
-245.934	-232.215	-665.41	-415880.	188.774	-4.1348
-6.81927	4.91549	-26.1317	-18540.	-4.1348	.129825

SIMPLE CORRELATION MATRIX

1	.369585	.961267	.963181	-.246679	-.260821
.369585	1	.325804	.31001	-.963253	.777515
.961267	.325804	1	.993513	-.187526	-.200823
.963181	.31001	.993513	1	-.18044	-.306738
-.246679	-.963253	-.187526	-.18044	1	-.835225
-.260821	.777515	-.200823	-.306738	-.835225	1

HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 2
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1,6
VARIABLES ARE 1 6

NORMAL EQUATION MATRIX
5265.39

RIGHT HAND SIDE VECTOR
-6.81927

INVERSE OF THE MATRIX
1.89920E-4

1 REG COEF-1.29511E-3
INTERCEPT 1.48697

ANALYSIS OF VARIANCE

VARIATION	DF	SS	MS	F TEST
TOTAL	10	.129825		
REGRESSION	1	8.83173E-3	8.83173E-3	.656941
ERROR	9	.120993	1.34437E-2	

MULTIPLE CORRELATION = R .260821
MULTIPLE CORR COEF = R*R = 6.80278E-2

HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 2
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 2,6
VARIABLES ARE 2 6

NORMAL EQUATION MATRIX
307.862

RIGHT HAND SIDE VECTOR
4.91549

INVERSE OF THE MATRIX
3.24821E-3

2 REG COEF 1.59665E-2
INTERCEPT 1.25264

ANALYSIS OF VARIANCE

VARIATION	DF	SS	MS	F TEST
TOTAL	10	.129825		
REGRESSION	1	7.84832E-2	7.84832E-2	13.7577
ERROR	9	.051342	5.70467E-3	

MULTIPLE CORRELATION = R .777515
MULTIPLE CORR COEF = R*R = .60453

HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 2
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 3,6
VARIABLES ARE 3 6

NORMAL EQUATION MATRIX
66697.6

RIGHT HAND SIDE VECTOR
-26.1317

INVERSE OF THE MATRIX
1.49930E-5

3 REG COEF-3.91793E-4
INTERCEPT 1.49646

ANALYSIS OF VARIANCE

VARIATION	DF	SS	MS	F TEST
TOTAL	10	.129825		
REGRESSION	1	1.02382E-2	1.02382E-2	.770518
ERROR	9	.119587	1.32874E-2	

MULTIPLE CORRELATION = R .280823
MULTIPLE CORR COEF = R*R = 7.88615E-2

HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 2
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 4,6
VARIABLES ARE 4 6

NORMAL EQUATION MATRIX
2.01402E+10

RIGHT HAND SIDE VECTOR
-18540.

INVERSE OF THE MATRIX
3.55363E-11

4 REG COEF-6.58845E-7
INTERCEPT 1.44169

ANALYSIS OF VARIANCE

VARIATION	DF	SS	MS	F TEST
TOTAL	10	.129825		
REGRESSION	1	.012215	.012215	.934741
ERROR	9	.11761	1.30673E-2	

MULTIPLE CORRELATION = R .306738
MULTIPLE CORR COEF = R*R = 9.40881E-2

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 2
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 5,6
 VARIABLES ARE 5  6

NORMAL EQUATION MATRIX
 108.774

RIGHT HAND SIDE VECTOR
-4.1348

INVERSE OF THE MATRIX
 5.29733E-3

 5 REG COEF-2.19034E-2
  INTERCEPT 1.96877

ANALYSIS OF VARIANCE
VARIATION        DF            SS              MS             F TEST
TOTAL            10            .129825
REGRESSION       1             9.05662E-2      9.05662E-2      20.762
ERROR            9             .039259         4.36211E-3

MULTIPLE CORRELATION = R .835225
MULTIPLE CORR  COEF  = R*R = .697601
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 3
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1,2,6
 VARIABLES ARE 1  2  6

NORMAL EQUATION MATRIX
 5265.39        470.552

 470.552        307.862

RIGHT HAND SIDE VECTOR
-6.81927        4.91549

INVERSE OF THE MATRIX
 2.19965E-4    -3.36205E-4

-3.36205E-4     3.76200E-3

 1 REG COEF-3.15262E-3
 2 REG COEF 2.07851E-2
  INTERCEPT 1.487

ANALYSIS OF VARIANCE
VARIATION        DF            SS              MS             F TEST
TOTAL            10            .129025
REGRESSION       2             .123468         6.18338E-2      80.3345
ERROR            8             6.15763E-3      7.69704E-4

MULTIPLE CORRELATION = R .975997
MULTIPLE CORR  COEF  = R*R = .95257
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 3
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1,3,6
 VARIABLES ARE 1  3  6

NORMAL EQUATION MATRIX
 5265.37        18014.2

 18014.2        66697.6

RIGHT HAND SIDE VECTOR
-6.81927        -26.1317

INVERSE OF THE MATRIX
 2.50008E-3     -6.75238E-4

-6.75238E-4      1.97366E-4

 1 REG COEF 5.96416E-4
 3 REG COEF-5.52977E-4
  INTERCEPT 1.49485

ANALYSIS OF VARIANCE
VARIATION       DF          SS             MS            F TEST
TOTAL           10          .129025
REGRESSION      2           1.03605E-2     5.19024E-3     .347625
ERROR           8           .119445        1.49306E-2

MULTIPLE CORRELATION = R .282767
MULTIPLE CORR  COEF  = R*R = 7.99574E-2
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 3
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1,4,6
 VARIABLES ARE 1  4  6

NORMAL EQUATION MATRIX
 5265.39        1.17243E+7

 1.17243E+7     2.81402E+10

RIGHT HAND SIDE VECTOR
-6.81927        -10540.

INVERSE OF THE MATRIX
 2.62747E-3     -1.09471E-6

-1.09471E-6      4.91633E-10

 1 REG COEF 2.37844E-3
 4 REG COEF-1.64579E-6
  INTERCEPT 1.33652

ANALYSIS OF VARIANCE
VARIATION       DF          SS             MS            F TEST
TOTAL           10          .129825
REGRESSION      2           .014368        7.18401E-3     .497778
ERROR           8           .115457        1.44321E-2

MULTIPLE CORRELATION = R .332674
MULTIPLE CORR  COEF  = R*R = .110672
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 3
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1,5,6
 VARIABLES ARE 1  5  6

NORMAL EQUATION MATRIX
 5265.39        -245.934

-245.934         188.774

RIGHT HAND SIDE VECTOR
-6.01927        -4.1348

INVERSE OF THE MATRIX
 2.02225E-4      2.63458E-4

 2.63458E-4      5.64056E-3

 1 REG COEF-2.46837E-3
 5 REG COEF-2.51192E-2
  INTERCEPT 2.26837

ANALYSIS OF VARIANCE
VARIATION        DF               SS              MS              F TEST
TOTAL            10               .129825
REGRESSION       2                .120695         6.03477E-2       52.8795
ERROR            8                9.12984E-3      1.14123E-3

MULTIPLE CORRELATION = R .964197
MULTIPLE CORR  COEF  = R*R = .929676
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 3
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 2,3,6
 VARIABLES ARE 2  3  6

NORMAL EQUATION MATRIX
 307.062         1476.35

 1476.35         66697.6

RIGHT HAND SIDE VECTOR
 4.91547        -26.1317

INVERSE OF THE MATRIX
 3.63394E-3     -8.04373E-5

-8.04373E-5      1.67735E-5

 2 REG COEF 1.99646E-2
 3 REG COEF-8.33708E-4
  INTERCEPT 1.47935

ANALYSIS OF VARIANCE
VARIATION        DF               SS              MS              F TEST
TOTAL            10               .129025
REGRESSION       2                .119922         5.99607E-2       48.4362
ERROR            8                9.90340E-3      1.23793E-3

MULTIPLE CORRELATION = R .961102
MULTIPLE CORR  COEF  = R*R = .923717
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 3
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 2,4,6
 VARIABLES ARE 2  4  6

NORMAL EQUATION MATRIX
 307.862        912468.

 912468.        2.81402E+10

RIGHT HAND SIDE VECTOR
 4.91547        -10540.

INVERSE OF THE MATRIX
 3.59357E-3     -1.16524E-7

-1.16524E-7      3.93147E-11

 2 REG COEF 1.98245E-2
 4 REG COEF-1.30167E-6
  INTERCEPT 1.3538

ANALYSIS OF VARIANCE
VARIATION       DF               SS              MS             F TEST
TOTAL           10               .129825
REGRESSION      2                .12158          6.07901E-2      58.9831
ERROR           8                8.24508E-3      1.03064E-3

MULTIPLE CORRELATION = R .967725
MULTIPLE CORR  COEF  = R*R = .936491
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 3
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 2,5,6
 VARIABLES ARE 2  5  6

NORMAL EQUATION MATRIX
 307.862        -232.215

-232.215         108.774

RIGHT HAND SIDE VECTOR
 4.91549        -4.1348

INVERSE OF THE MATRIX
 4.50238E-2     5.53846E-2

 5.53846E-2     7.34269E-2

 2 REG COEF-7.69045E-3
 5 REG COEF-3.13634E-2
  INTERCEPT 2.20418

ANALYSIS OF VARIANCE
VARIATION       DF               SS              MS             F TEST
TOTAL           10               .129825
REGRESSION      2                9.18798E-2      4.59399E-2      9.68549
ERROR           8                3.79454E-2      4.74317E-3

MULTIPLE CORRELATION = R .841261
MULTIPLE CORR  COEF  = R*R = .70772
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 3
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 3,4,6
 VARIABLES ARE 3  4  6

NORMAL EQUATION MATRIX
 66697.6        4.30420E+7

 4.30420E+7     2.81402E+10

RIGHT HAND SIDE VECTOR
-26.1317        -18540.

INVERSE OF THE MATRIX
 1.15944E-3     -1.77343E-6

-1.77343E-6      2.74809E-9

 3 REG COEF 2.58130E-3
 4 REG COEF-4.60709E-6
  INTERCEPT 1.0414

ANALYSIS OF VARIANCE
VARIATION        DF           SS            MS           F TEST
TOTAL            10           .129825
REGRESSION       2            1.79619E-2    8.98093E-3    .642278
ERROR            8            .111863       1.39829E-2

MULTIPLE CORRELATION = R .37196
MULTIPLE CORR  COEF  = R*R = .130354
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 3
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 3,5,6
 VARIABLES ARE 3  5  6

NORMAL EQUATION MATRIX
 66697.6        -665.41

-665.41          188.774

RIGHT HAND SIDE VECTOR
-26.1317        -4.1348

INVERSE OF THE MATRIX
 1.55395E-5      5.47751E-5

 5.47751E-5      5.49041E-3

 3 REG COEF-6.32557E-4
 5 REG COEF-2.41331E-2
  INTERCEPT 2.22451

ANALYSIS OF VARIANCE
VARIATION        DF           SS            MS           F TEST
TOTAL            10           .129825
REGRESSION       2            .116315       5.81577E-2    34.4388
ERROR            8            1.35098E-2    1.68873E-3

MULTIPLE CORRELATION = R .94654
MULTIPLE CORR  COEF  = R*R = .895930
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE O(ZERO) ? 3
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 4,5,6
 VARIABLES ARE 4  5  6

NORMAL EQUATION MATRIX
 2.81402E+10   -415880.

-415880.        180.774


RIGHT HAND SIDE VECTOR
-18540.         -4.1348


INVERSE OF THE MATRIX
 3.67323E-11     8.09232E-8

 8.09232E-8      5.47561E-3


 4 REG COEF-1.01562E-6
 5 REG COEF-2.41409E-2
  INTERCEPT 2.13146


ANALYSIS OF VARIANCE
VARIATION         DF              SS             MS            F TEST
TOTAL             10             .129825
REGRESSION        2              .118647        5.93237E-2     42.4579
ERROR             8             1.11779E-2      1.39723E-3

MULTIPLE CORRELATION = R .955981
MULTIPLE CORR  COEF  = R*R = .913901
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE O(ZERO) ? 4
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1,2,3,6
 VARIABLES ARE 1  2  3  6

NORMAL EQUATION MATRIX
 5265.39        470.552        18014.2

 470.552        307.862        1476.35

 18014.2        1476.35        66697.6


RIGHT HAND SIDE VECTOR
-6.81927        4.91549       -26.1317


INVERSE OF THE MATRIX
 2.62295E-3    -6.84448E-4    -6.93275E-4

-6.84448E-4     3.81255E-3     1.00470E-4

-6.93275E-4     1.00470E-4     2.00014E-4


 1 REG COEF-3.13457E-3
 2 REG COEF 2.07035E-2
 3 REG COEF-5.20648E-6
  INTERCEPT 1.48707


ANALYSIS OF VARIANCE
VARIATION         DF              SS             MS             F TEST
TOTAL             10             .129825
REGRESSION        3              .123668        4.12226E-2      46.8629
ERROR             7             6.15750E-3      8.79643E-4


MULTIPLE CORRELATION = R .975997
MULTIPLE CORR  COEF  = R*R = .952571
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 4
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1,2,4,6
 VARIABLES ARE 1  2  4  6

NORMAL EQUATION MATRIX
 5265.39        470.552        1.17243E+7

 470.552        307.862        912468.

 1.17243E+7     912468.        2.81402E+10

RIGHT HAND SIDE VECTOR
-6.81927        4.91549       -18540.

INVERSE OF THE MATRIX
 2.84707E-3    -9.24717E-4    -1.15621E-6

-9.24717E-4     3.89391E-3     2.59010E-7

-1.15621E-6     2.59010E-7     5.00881E-10

 1 REG COEF-2.52412E-3
 2 REG COEF 2.06443E-2
 4 REG COEF-2.76606E-7
  INTERCEPT 1.46178

ANALYSIS OF VARIANCE
VARIATION        DF            SS             MS              F TEST
TOTAL            10            .127825
REGRESSION       3             .123818        4.12726E-2      48.0931
ERROR            7             6.00720E-3     8.50182E-4

MULTIPLE CORRELATION = R .97659
MULTIPLE CORR  COEF  = R*R = .953728
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 4
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1,2,5,6
 VARIABLES ARE 1  2  5  6

NORMAL EQUATION MATRIX
 5265.39        470.552       -245.934

 470.552        307.862       -232.215

-245.934       -232.215        188.774

RIGHT HAND SIDE VECTOR
-6.81927        4.91549       -4.1348

INVERSE OF THE MATRIX
 2.72192E-4    -2.05914E-3    -2.17838E-3

-2.05914E-3     6.06013E-2     7.18641E-2

-2.17838E-3     7.18641E-2     9.00607E-2

 1 REG COEF-2.97067E-3
 2 REG COEF 1.47820E-2
 5 REG COEF-7.50897E-3
  INTERCEPT 1.72307

ANALYSIS OF VARIANCE
VARIATION        DF            SS             MS              F TEST
TOTAL            10            .127825
REGRESSION       3             .124301        4.14338E-2      52.5069
ERROR            7             5.52378E-3     7.89111E-4

MULTIPLE CORRELATION = R .978495
MULTIPLE CORR  COEF  = R*R = .957452
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE O(ZERO) ? 4
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1,3,4,6
 VARIABLES ARE 1  3  4  6

NORMAL EQUATION MATRIX
 5265.39          18014.2          1.17243E+7

 18014.2          66697.6          4.30420E+7

 1.17243E+7       4.30420E+7       2.81402E+10

RIGHT HAND SIDE VECTOR
-6.81927         -26.1317          -18540.

INVERSE OF THE MATRIX
 2.68135E-3      -2.52494E-4       -7.30952E-7

-2.52494E-4       1.18322E-3       -1.70460E-6

-7.30952E-7      -1.70460E-6        2.94735E-9

 1 REG COEF 1.86508E-3
 3 REG COEF 2.40567E-3
 4 REG COEF-5.11552E-6
  INTERCEPT .986164

ANALYSIS OF VARIANCE
VARIATION        DF             SS              MS              F TEST
TOTAL            10             .129825
REGRESSION       3              1.92592E-2      6.41972E-3      .406436
ERROR            7              .110566         1.57951E-2

MULTIPLE CORRELATION = R .385158
MULTIPLE CORR  COEF  = R*R = .148347
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE O(ZERO) ? 4
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1,3,5,6
 VARIABLES ARE 1  3  5  6

NORMAL EQUATION MATRIX
 5265.39          18014.2          -245.934

 18014.2          66697.6          -665.41

-245.934         -665.41            188.774

RIGHT HAND SIDE VECTOR
-6.81927         -26.1317          -4.1348

INVERSE OF THE MATRIX
 2.66017E-3      -7.08830E-4        9.67105E-4

-7.08830E-4       2.04415E-4       -2.02920E-4

 9.67105E-4      -2.02920E-4        .005042

 1 REG COEF-3.61631E-3
 3 REG COEF 3.31044E-4
 5 REG COEF-2.54470E-2
  INTERCEPT 2.27309

ANALYSIS OF VARIANCE
VARIATION        DF             SS              MS              F TEST
TOTAL            10             .129825
REGRESSION       3              .121231         4.04105E-2      32.9163
ERROR            7              8.59372E-3      1.22767E-3

MULTIPLE CORRELATION = R .966336
MULTIPLE CORR  COEF  = R*R = .933805
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 4
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1,4,5,6
 VARIABLES ARE 1  4  5  6

NORMAL EQUATION MATRIX
 5265.39        1.17243E+7     -245.934

 1.17243E+7     2.81402E+10    -415880.

-245.934       -415880.         188.774

RIGHT HAND SIDE VECTOR
-6.81727       -18540.         -4.1348

INVERSE OF THE MATRIX
 2.84346E-3    -1.16778E-6      1.13133E-3

-1.16790E-6     5.16480E-10    -3.83781E-7

 1.13133E-3    -3.83781E-7      5.92573E-3

 1 REG COEF-2.41306E-3
 4 REG COEF-2.41072E-8
 5 REG COEF-2.51013E-2
  INTERCEPT 2.26564

ANALYSIS OF VARIANCE
VARIATION         DF              SS              MS              F TEST
TOTAL             10              .129825
REGRESSION        3               .120696         4.02322E-2       30.8505
ERROR             7               9.12871E-3      1.30410E-3

MULTIPLE CORRELATION = R .964202
MULTIPLE CORR  COEF  = R*R = .929685
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 4
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 2,3,4,6
 VARIABLES ARE 2  3  4  6

NORMAL EQUATION MATRIX
 307.862        1476.35         912468.

 1476.35        66697.6         4.30420E+7

 912468.        4.30420E+7      2.81402E+10

RIGHT HAND SIDE VECTOR
 4.91549       -26.1317        -18540.

INVERSE OF THE MATRIX
 3.69375E-3    -3.45535E-4      4.08742E-7

-3.45535E-4     1.19176E-3     -1.81166E-6

 4.08742E-7    -1.81166E-6      2.79332E-9

 2 REG COEF 1.96079E-2
 3 REG COEF 7.47067E-4
 4 REG COEF-2.43733E-6
  INTERCEPT 1.23891

ANALYSIS OF VARIANCE
VARIATION         DF              SS              MS              F TEST
TOTAL             10              .129825
REGRESSION        3               .122048         4.06828E-2       36.6192
ERROR             7               7.77678E-3      1.11097E-3

MULTIPLE CORRELATION = R .969587
MULTIPLE CORR  COEF  = R*R = .940098
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 4
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 2,3,5,6
 VARIABLES ARE 2  3  5  6

NORMAL EQUATION MATRIX
 307.862        1476.35        -232.215

 1476.35        66697.6        -665.41

-232.215        -665.41         188.774

RIGHT HAND SIDE VECTOR
 4.91549        -26.1317       -4.1348

INVERSE OF THE MATRIX
 6.45737E-2     -6.60084E-4     7.71066E-2

-6.60084E-4      2.22870E-5    -7.33422E-4

 7.71066E-2     -7.33422E-4     9.75624E-2

 2 REG COEF 1.58399E-2
 3 REG COEF-7.94475E-4
 5 REG COEF-5.21892E-3
  INTERCEPT 1.64033

ANALYSIS OF VARIANCE
VARIATION       DF            SS              MS              F TEST
TOTAL           10            .129825
REGRESSION      3             .120201         .040067          29.1417
ERROR           7             9.62430E-3      1.37490E-3

MULTIPLE CORRELATION = R .96222
MULTIPLE CORR  COEF  = R*R = .925867
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 4
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 2,4,5,6
 VARIABLES ARE 2  4  5  6

NORMAL EQUATION MATRIX
 307.062        912460.        -232.215

 912468.        2.81402E+10    -415080.

-232.215        -415080.        180.774

RIGHT HAND SIDE VECTOR
 4.91549        -18540.        -4.1348

INVERSE OF THE MATRIX
 6.13223E-2     -9.02997E-7     7.34443E-2

-9.02997E-7      5.00293E-11   -1.00058E-6

 7.34443E-2     -1.00058E-6     9.34383E-2

 2 REG COEF 1.44920E-2
 4 REG COEF-1.22903E-6
 5 REG COEF-6.78316E-3
  INTERCEPT 1.57125

ANALYSIS OF VARIANCE
VARIATION       DF            SS              MS              F TEST
TOTAL           10            .129025
REGRESSION      3             .122073         4.06908E-2       36.7404
ERROR           7             7.75266E-3      1.10752E-3

MULTIPLE CORRELATION = R .969682
MULTIPLE CORR  COEF  = R*R = .940284
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 4
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 3,4,5,6
 VARIABLES ARE 3  4  5  6

NORMAL EQUATION MATRIX
 66697.6        4.30420E+7     -665.41

 4.30420E+7     2.81402E+10    -415880.

-665.41        -415880.         188.774

RIGHT HAND SIDE VECTOR
-26.1317       -18540.         -4.1348

INVERSE OF THE MATRIX
 1.16579E-3    -1.78038E-6      1.87022E-4

-1.78038E-6     2.75570E-9     -2.04694E-7

 1.87022E-4    -2.04694E-7      5.50561E-3

 3 REG COEF 1.77090E-3
 4 REG COEF-3.72011E-6
 5 REG COEF-2.38568E-2
  INTERCEPT 1.84073

ANALYSIS OF VARIANCE
VARIATION         DF              SS              MS              F TEST
TOTAL             10              .129825
REGRESSION        3               .121337         4.04458E-2       33.3563
ERROR             7               8.48777E-3      1.21254E-3

MULTIPLE CORRELATION = R .966758
MULTIPLE CORR  COEF  = R*R = .934622
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 5
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1 2 3 4 6
 VARIABLES ARE 1  2  3  4  6

NORMAL EQUATION MATRIX
 5265.39        470.552         18014.2         1.17243E+7

 470.552        307.862         1476.35         912468.

 18014.2        1476.35         66697.6         4.30420E+7

 1.17243E+7     912468.         4.30420E+7      2.81402E+10

RIGHT HAND SIDE VECTOR
-6.81927        4.91549        -26.1317        -18540.

INVERSE OF THE MATRIX
 2.87661E-3    -8.79631E-4     -1.88595E-4     -8.81518E-7

-8.79631E-4     3.96273E-3     -2.87865E-4      6.78298E-7

-1.88595E-4    -2.87865E-4      1.20413E-3     -1.75387E-6

-8.81518E-7     6.78298E-7     -1.75387E-6      3.06345E-9

 1 REG COEF-2.66853E-3
 2 REG COEF 2.04239E-2
 3 REG COEF 9.22020E-4
 4 REG COEF-1.61957E-6
  INTERCEPT 1.32616

ANALYSIS OF VARIANCE
VARIATION         DF              SS              MS              F TEST
TOTAL             10              .129825
REGRESSION        4               .124524         .031131          35.2342
ERROR             6               5.30127E-3      8.83545E-4

MULTIPLE CORRELATION = R .97937
MULTIPLE CORR  COEF  = R*R = .959166
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 5
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1 2 3 5 6
 VARIABLES ARE 1  2  3  5  6

NORMAL EQUATION MATRIX
 5265.39         470.552        18014.2        -245.934

 470.552         307.862        1476.35        -232.215

 18014.2         1476.35        66697.6        -665.41

-245.934        -232.215       -665.41          188.774

RIGHT HAND SIDE VECTOR
-6.81927         4.91549       -26.1317        -4.1348

INVERSE OF THE MATRIX
 2.68447E-3      1.25832E-3    -7.28168E-4      2.47848E-3

 1.25832E-3      6.51636E-2    -1.00141E-3      7.82684E-2

-7.28168E-4     -1.00141E-3     2.19804E-4     -1.40571E-3

 2.47848E-3      7.82684E-2    -1.40571E-3      9.98507E-2

 1 REG COEF-3.34067E-3
 2 REG COEF .014274
 3 REG COEF 1.11688E-4
 5 REG COEF-8.30325E-3
  INTERCEPT 1.7437

ANALYSIS OF VARIANCE
VARIATION        DF            SS              MS              F TEST
TOTAL            10            .129825
REGRESSION       4             .124358         3.10895E-2      34.1204
ERROR            6             5.46703E-3      9.11171E-4

MULTIPLE CORRELATION = R .978718
MULTIPLE CORR  COEF  = R*R = .957889
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 5
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1 2 4 5 6
 VARIABLES ARE 1  2  4  5  6

NORMAL EQUATION MATRIX
 5265.39         470.552        1.17243E+7     -245.934

 470.552         307.862        912468.        -232.215

 1.17243E+7      912468.        2.81402E+10    -415880.

-245.934        -232.215       -415880.         188.774

RIGHT HAND SIDE VECTOR
-6.81927         4.91549       -18540.         -4.1348

INVERSE OF THE MATRIX
 2.84706E-3     -5.86389E-4    -1.16164E-6      4.31254E-4

-5.86309E-4      .061443       -6.63910E-7      7.33556E-2

-1.16164E-6     -6.63910E-7     5.23662E-10    -1.17641E-6

 4.31254E-4      7.33556E-2    -1.17641E-6      9.35035E-2

 1 REG COEF-2.55719E-3
 2 REG COEF 1.50191E-2
 4 REG COEF-1.86394E-7
 5 REG COEF-7.17023E-3
  INTERCEPT 1.69305

ANALYSIS OF VARIANCE
VARIATION        DF            SS              MS              F TEST
TOTAL            10            .129825
REGRESSION       4             .124368         3.10919E-2      34.183
ERROR            6             5.45743E-3      9.09572E-4

MULTIPLE CORRELATION = R .978756
MULTIPLE CORR  COEF  = R*R = .957963
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 5
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1 3 4 5 6
 VARIABLES ARE 1  3  4  5  6

NORMAL EQUATION MATRIX
 5263.39       18014.2       1.17243E+7    -245.934

 18014.2       66697.6       4.30420E+7    -665.41

 1.17243E+7    4.30420E+7    2.81402E+10   -415880.

-245.934      -665.41       -415880.        188.774

RIGHT HAND SIDE VECTOR
-6.81927      -26.1317      -18540.       -4.1348

INVERSE OF THE MATRIX
 2.88982E-3   -2.34344E-4   -8.29130E-7    1.11218E-3

-2.34344E-4    1.18480E-3   -1.71314E-6    9.68321E-5

-8.29130E-7   -1.71314E-6    2.99359E-9   -5.23794E-7

 1.11218E-3    9.68321E-5   -5.23794E-7    5.93365E-3

 1 REG COEF-2.80919E-3
 3 REG COEF 1.99871E-3
 4 REG COEF-2.91412E-6
 5 REG COEF-2.49379E-2
  INTERCEPT 1.9605

ANALYSIS OF VARIANCE
VARIATION      DF            SS            MS            F TEST
TOTAL          10            .129825
REGRESSION     4             .124068       3.10171E-2    32.3265
ERROR          6             5.75696E-3    9.59494E-4

MULTIPLE CORRELATION = R .977577
MULTIPLE CORR  COEF  = R*R = .955656
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MRE, PLEASE TYPE 0(ZERO) ? 5
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 2 3 4 5 6
 VARIABLES ARE 2  3  4  5  6

NORMAL EQUATION MATRIX
 307.062       1476.35       912468.       -232.215

 1476.35       66697.6       4.30420E+7    -665.41

 912468.       4.30420E+7    2.81402E+10   -415880.

-232.215      -665.41       -415880.        188.774

RIGHT HAND SIDE VECTOR
 4.91549      -26.1317      -18540.       -4.1348

INVERSE OF THE MATRIX
 7.20253E-2   -3.82822E-3    4.78579E-6    8.56489E-2

-3.82822E-3    1.36921E-3   -2.03475E-6   -4.36530E-3

 4.78579E-6   -2.03475E-6    3.07370E-9    5.48633E-6

 8.56489E-2   -4.36530E-3    5.48633E-6    .107355

 2 REG COEF 1.12071E-2
 3 REG COEF 1.17524E-3
 4 REG COEF-2.97545E-6
 5 REG COEF-1.05299E-2
  INTERCEPT 1.51063

ANALYSIS OF VARIANCE
VARIATION      DF            SS            MS            F TEST
TOTAL          10            .129825
REGRESSION     4             .123081       3.07703E-2    27.3759
ERROR          6             6.74396E-3    1.12399E-3

MULTIPLE CORRELATION = R .97368
MULTIPLE CORR  COEF  = R*R = .948054
```

```
HOW MANY VARIABLES (INCLUDES A DEPENDENT VARIABLE)
IF NO MORE, PLEASE TYPE 0(ZERO) ? 6
WHAT ARE THEY. (LIST THE DEPENDENT VARIABLE LAST) ? 1,2,3,4,5,6
 VARIABLES ARE 1  2  3  4  5  6

NORMAL EQUATION MATRIX
 5265.39        470.552        18014.2        1.17243E+7     -245.934

 470.552        307.862        1476.35        912468.        -232.215

 18014.2        1476.35        66697.6        4.30420E+7     -665.41

 1.17243E+7     912468.        4.30420E+7     2.81402E+10    -415880.

-245.934       -232.215       -665.41        -415880.         188.774

RIGHT HAND SIDE VECTOR
-6.81927        4.91549       -26.1317       -18540.        -4.1348

INVERSE OF THE MATRIX
 2.88989E-3     7.07498E-5    -2.38121E-4    -8.24435E-7      1.19658E-3

 7.07498E-5     .072027       -3.83406E-3     4.76555E-6      8.56783E-2

-2.38121E-4    -3.83406E-3     1.38889E-3    -1.96682E-6     -4.46390E-3

-8.24435E-7     4.76555E-6    -1.96682E-6     3.30889E-9      5.14497E-6

 1.19658E-3     8.56783E-2    -4.46390E-3     5.14497E-6      .107851

 1 REG COEF-2.79821E-3
 2 REG COEF 1.11384E-2
 3 REG COEF 1.40580E-3
 4 REG COEF-2.17717E-6
 5 REG COEF-1.16885E-2
  INTERCEPT 1.63201

ANALYSIS OF VARIANCE
VARIATION       DF           SS             MS            F TEST
TOTAL           10           .129825
REGRESSION      5            .125791        2.51581E-2     31.1787
ERROR           5            4.03451E-3     8.06902E-4

MULTIPLE CORRELATION = R .984339
MULTIPLE CORR  COEF  = R*R = .968924
```

APPENDIX 4

Program listing for computing the expected S/R travel time by complete enumeration and the conventional method

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION TIME(50,350)
C
C     INITIALIZE THE SYSTEM PARAMETERS
C
      HEHT=50.0
      WTH=50.0
      HEHT=HEHT/12.0
      WTH=WTH/12.0
      HVEL=240.0
      VVEL=60.0
      ICOL=34
      NOL=3
C
C     INITIALIZE PROGRAM VARIABLES
C     CUMT=CUMULATIVE TOTAL TRAVEL TIME FOR SC
C     CUMS=CUMULATIVE TOTAL OF SQUARE OF TRAVEL TIME FOR SC
C     TOT=CUMULATIVE TOTAL TRAVEL TIME FOR DC
C     TOS=CUMULATIVE TOTAL OF SQUARE OF TRAVEL TIME FOR DC
C
      CUMT=0.0
      CUMS=0.0
      TOT=0.0
      TOS=0.0
      NSL=ICOL*NOL
      PHET=-HEHT/2.0
C
C     DETERMINE THE TRAVEL TIME TO EACH OPENING
C     STORE THEM IN TIME(I,J)  ADD THE TRAVEL
C     TIME AND ITS SQUARE TO CUMT AND CUMS,RESPECTIVELY
C
      DO 13 K1=1,NOL
      PHET=PHET+HEHT
      FWT=WTH/2.0
      DO 13 K2=1,ICOL
      TH=FWT/HVEL
      TV=PHET/VVEL
      TIME(K1,K2)=TH
      IF(TV.GT.TH)TIME(K1,K2)=TV
      FWT=FWT+WTH
      CUMT=CUMT+TIME(K1,K2)
13    CUMS=CUMS+(TIME(K1,K2)**2)
      NS1=NSL-1
C
C     DETERMINE TOT & TOS BY ENUMERATION
C     OVER TOTAL NUMBER OF OPENINGS
C
      DO 12 LM=1,NS1
      IX1=((LM-0.00001)/ICOL)+1.0
      IY1=LM-((IX1-1)*ICOL)
      LT=LM+1
      DO 12 KK=LT,NSL
      IX2=((KK-0.00001)/ICOL)+1.0
      IY2=KK-((IX2-1)*ICOL)
      SLT=ABS((IY1-IY2)*WTH)/HVEL
      VTM=ABS((IX1-IX2)*HEHT)/VVEL
      IF(VTM.GT.SLT)SLT=VTM
      ALLT=TIME(IX1,IY1)+TIME(IX2,IY2)+SLT
      TOT=TOT+ALLT
      TOS=TOS+(ALLT**2)
12    CONTINUE
      ETS=(2.0/NSL)*CUMT
      VTS=((4.0/NSL)*CUMS)-ETS**2
      DEN=2.0/(NSL**2-NSL)
      ETD=DEN*TOT
      VTD=(DEN*TOS)-ETD**2
      WRITE(6,32)ETS,VTS,ETD,VTD
32    FORMAT(3X,'SINGLE',2X,2F9.4,2X,'DUAL',2X,2F9.4)
```

```
C
C     SC AND DC TRAVEL TIME AND THEIR CORRESPONDING
C     VARIANCE HAS BEEN COMPUTED. FOLLOWING IS FOR THE
C     CONVENTIONAL METHOD OF COMPUTING EXP. SC & DC
C     TRAVEL TIMES.
C
      HDT=WTH*ICOL
      VDT=HEHT*NOL
      IH1=((HDT/2.0)-0.000001)+1.0
      IH2=((0.75*HDT)-0.000001)+1.0
      IV1=((VDT/2.0)-0.000001)+1.0
      IV2=((0.75*VDT)-0.000001)+1.0
      SIT=IH1/HVEL
      CT=IV1/VVEL
      IF(CT.GT.SIT)SIT=CT
      TBT=(IH2-IH1)/HVEL
      CTB=(IV2-IV1)/VVEL
      IF(CTB.GT.TBT)TBT=CTB
      SC=SIT*2.0
      DC=(SIT+TBT)*2.0
      WRITE(6,15)SC,DC
15    FORMAT(3X,'CONVENTIONAL SINGLE & DUAL COMM.',1X,2F9.4)
      STOP
      END
```

APPENDIX 5

Program listing for simulating dual and single command trips over a continuous normalized rack

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE9)
C
C      FILE NAME: SIMB
C      THIS PROGRAM SIMULATES A RACK WITH A GIVEN
C      SHAPE FACTOR,B IN ORDER TO FIND THE EXPECTED
C      MEAN AND VARIANCE OF A DUAL CYCLE. IT IS ALSO
C      USED TO PLOT HISTOGRAPHS OF SINGLE AND DUAL CYCLE
C      MEAN TRAVEL TIMES.
C
      DIMENSION SC(1000),DC(1000)
      BB=1.000
      DCT=0.0
      DCS=0.0
C
C      START SYSTEM SIMULATION.  NO. OF TRIPS=1000
C      LOCATION OF THE FIRST SLOT= (R1,R2)
C      LOCATION OF THE SECOND SLOT= (R3,R4)
C      'DCT' STORES CUMULATIVE DUAL CYCLE TRAVEL TIMES
C      'DCS' STORES CUMULATIVE SQUARE OF DUAL CYCLE TRAVEL TIMES
C
      DO 2 K2=1,1000
      R1=RANF(X)
      R2=BB*RANF(X)
      R3=RANF(X)
      R4=BB*RANF(X)
      SC(K2)=R1
      IF(R2.GT.R1)SC(K2)=R2
      TBT=ABS(R3-R1)
      T1=ABS(R4-R2)
      IF(T1.GT.TBT)TBT=T1
      TBK=R3
      IF(R4.GT.R3)TBK=R4
      DC(K2)=SC(K2)+TBT+TBK
      SC(K2)=SC(K2)*2.0
      DCS=DCS+(DC(K2)**2)
      DCT=DCT+DC(K2)
2     CONTINUE
C
C      END OF SIMULATION. STORE BOTH SINGLE AND DUAL CYCLE
C      TRAVEL TIMES ON TAPE 9.
C
      DO 3 K3=1,1000
3     WRITE(9,*)SC(K3)
      DO 4 K4=1,1000
4     WRITE(9,*)DC(K4)
      RMEAN=DCT/1000.0
      VAR=(DCS-(1000.0*RMEAN**2))/999.0
C
C      PRINT SHAPE FACTOR, SIMULATED MEAN AND VARIANCE OF
C      DUAL CYCLE TRAVEL TIME.
C
      WRITE(6,7)BB,RMEAN,VAR
7     FORMAT(/7X,'SHAPE FACTOR=',F5.2,2X,'MEAN DC T.TIME',
     *F8.4,2X,'VAR. OF DC T.TIME',F7.4)
      STOP
      END
```

APPENDIX 6

Program listing for finding the expected mean and variance of travel time by enumeration under dedicated storage

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C      FILE NAME: DEDIA
C      THIS PROGRAM COMPUTES THE MEAN AND VARIANCE OF
C      THE SINGLE AND DUAL COMMAND TRAVEL TIMES FOR
C      A GIVEN RACK. THE SUBROUTINE IT USES IS THE
C      THE SAME SUBROUTINE USED BY <TEMP>.
C
      DIMENSION IA(10),C(10),TNC(10)
      COMMON HVEL,VVEL,NOPR
      HEHT=58.0/12.0
      WTH=56.0/12.0
      HVEL=240.0
      VVEL=60.0
      NOPR=5
      NOL=10
      ICOL=40
      DO 77 K77=1,NOPR
77    READ(5,*)C(K77),IA(K77)
      NOA=0
      CAPC=0.0
      DELT=0.0
      DO 2 K2=1,NOPR
      NOA=NOA+IA(K2)
2     CAPC=CAPC+C(K2)
      DO 3 K3=1,NOPR
3     TNC(K3)=C(K3)/CAPC
      DO 4 K4=1,NOPR
4     DELT=DELT+((TNC(K4)**2/IA(K4)**2)*IA(K4))
      DELT=(1.0-DELT)/2.0
      CALL TRAVEL(HEHT,WTH,IA,C,CAPC,TNC,DELT,NOL,ICOL,
     *ESC,VSC,EDC,VDC)
      WRITE(6,7)ESC,VSC,EDC,VDC,NOA
7     FORMAT(///4X,'TRUE SINGLE(VAR) & DUAL(VAR) TIMES',
     *1X,4F8.4,3X,I5)
      STOP
      END
      SUBROUTINE TRAVEL(HEHT,WTH,MIA,C,CAPC,TNC,DELT,NOL,ICOL,
     *ETS,VTS,EDT,VDT)
      DIMENSION MIA(10),C(10),TNC(10),TIME(50,300),IPRD(50,300)
      COMMON HVEL,VVEL,NOPR
      TOTM=0.0
      TOTS=0.0
      PHET=-HEHT/2.0
      DO 12 JM=1,NOL
      PHET=PHET+HEHT
      FWT=WTH/2.0
      DO 12 JC=1,ICOL
      TH=FWT/HVEL
      TV=PHET/VVEL
      TIME(JM,JC)=TH
      IF(TV.GT.TH)TIME(JM,JC)=TV
      FWT=FWT+WTH
12    IPRD(JM,JC)=0
C
```

```
      DO 16 NP=1,NOPR
      CUTM=0.0
      CUTS=0.0
      ILG=MIA(NP)
      DO 19 MN=1,ILG
      TMINI=10000.0
      LX=1
      LY=1
      DO 17 ILE=1,NOL
      DO 17 ICO=1,ICOL
      IF(IPRD(ILE,ICO).NE.0) GO TO 17
      IF(TIME(ILE,ICO).GE.TMINI) GO TO 17
      TMINI=TIME(ILE,ICO)
      LX=ILE
      LY=ICO
17    CONTINUE
      IPRD(LX,LY)=NP
      CUTM=CUTM+TIME(LX,LY)
      CUTS=CUTS+(TIME(LX,LY)*TIME(LX,LY))
19    CONTINUE
      TOTM=TOTM+((C(NP)/ILG)*CUTM)
      TOTS=TOTS+((C(NP)/ILG)*CUTS)
16    CONTINUE
      ETS=TOTM*(2.0/CAPC)
      VTS=((4.0/CAPC)*TOTS)-((4/(CAPC*CAPC))*TOTM*TOTM)
      STAM=0.0
      RTAM=0.0
      NSL=ICOL*NOL
      NS1=NSL-1
      DO 88 L6=1,NS1
      JPR=((L6-0.00001)/ICOL)+1.0
      JP=L6-((JPR-1)*ICOL)
      L7=L6+1
      DO 88 L8=L7,NSL
      KG=((L8-0.00001)/ICOL)+1.0
      KLM=L8-((KG-1)*ICOL)
      SLT=(ABS((KLM-JP)*WTH))/HVEL
      VTTM=(ABS((KG-JPR)*HEHT))/VVEL
      IF(VTTM.GT.SLT) SLT=VTTM
      TTAYM=TIME(JPR,JP)+TIME(KG,KLM)+SLT
      MA1=IPRD(JPR,JP)
      MA2=IPRD(KG,KLM)
      IF(MA1*MA2)49,88,49
49    BOL=MIA(MA1)*MIA(MA2)*DELT
      RTAM=RTAM+((TTAYM*TNC(MA1)*TNC(MA2))/BOL)
      STAM=STAM+((TTAYM*TTAYM*TNC(MA1)*TNC(MA2))/BOL)
88    CONTINUE
      EDT=RTAM
      VDT=STAM-(EDT*EDT)
      RETURN
      END
```

APPENDIX 7

Program listing for simulating dual command trips over a discrete rack under dedicated storage

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C        FILE NAME: DEDISIM
C        THIS PROGRAM IS SIMILAR TO <DEDIA>. THE SUBROUTINE
C        IT USES, HOWEVER, DOES NOT ENUMERATE OVER ALL
C        POSSIBLE DUAL CYCLES  INSTEAD, IT USES SIMULATION
C        TO COMPUTE THE EXPECTED MEAN AND VARIANCE OF DUAL
C        CYCLES. THE EXPECTED MEAN AND VARIANCE OF SINGLE CYCLES
C        ARE COMPUTED WHILE ALLOCATING THE PRODUCT CLASSES TO THE
C        OPENINGS. HENCE, SINGLE CYCLE IS FOUND ACCURATELY.
C
      DIMENSION IA(10),C(10),TNC(10)
      COMMON HVEL,VVEL,NOPR
      HEHT=58.0/12.0
      WTH=56.0/12.0
      HVEL=240.0
      VVEL=60.0
      NOPR=5
      NOL=10
      ICOL=40
      DO 77 K77=1,NOPR
77    READ(5,*)C(K77),IA(K77)
      NOA=0
      CAPC=0.0
      DELT=0.0
      DO 2 K2=1,NOPR
      NOA=NOA+IA(K2)
2     CAPC=CAPC+C(K2)
      DO 3 K3=1,NOPR
3     TNC(K3)=C(K3)/CAPC
      DO 4 K4=1,NOPR
4     DELT=DELT+((TNC(K4)**2/IA(K4)**2)*IA(K4))
      CALL TRAVEL(HEHT,WTH,IA,C,CAPC,TNC,DELT,NOL,ICOL,
     *ESC,VSC,EDC,VDC)
      WRITE(6,7)ESC,VSC,EDC,VDC,NOA
7     FORMAT(///4X,'SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES',
     *1X,4F8.4,3X,I5)
      STOP
      END
      SUBROUTINE TRAVEL(HEHT,WTH,MIA,C,CAPC,TNC,DELT,NOL,ICOL,
     *ETS,VTS,EDT,VDT)
      DIMENSION MIA(10),C(10),TNC(10),TIME(50,300),IPRD(50,300)
      COMMON HVEL,VVEL,NOPR
      TOTM=0.0
      TOTS=0.0
      PHET=-HEHT/2.0
      DO 12 JM=1,NOL
      PHET=PHET+HEHT
      FWT=WTH/2.0
      DO 12 JC=1,ICOL
      TH=FWT/HVEL
      TV=PHET/VVEL
      TIME(JM,JC)=TH
      IF(TV.GT.TH)TIME(JM,JC)=TV
      FWT=FWT+WTH
12    IPRD(JM,JC)=0
C
      DO 16 NP=1,NOPR
      CUTM=0.0
      CUTS=0.0
      ILG=MIA(NP)
```

```
      DO 19 MN=1,ILG
      TMINI=10000.0
      LX=1
      LY=1
      DO 17 ILE=1,NOL
      DO 17 ICO=1,ICOL
      IF(IPRD(ILE,ICO).NE.0) GO TO 17
      IF(TIME(ILE,ICO).GE.TMINI) GO TO 17
      TMINI=TIME(ILE,ICO)
      LX=ILE
      LY=ICO
17    CONTINUE
      IPRD(LX,LY)=NP
      CUTM=CUTM+TIME(LX,LY)
      CUTS=CUTS+(TIME(LX,LY)*TIME(LX,LY))
19    CONTINUE
      TOTM=TOTM+((C(NP)/ILG)*CUTM)
      TOTS=TOTS+((C(NP)/ILG)*CUTS)
16    CONTINUE
      ETS=TOTM*(2.0/CAPC)
      VTS=((4.0/CAPC)*TOTS)-((4/(CAPC*CAPC))*TOTM*TOTM)
      STAM=0.0
      RTAM=0.0
C
C      'WEIT' IS USED TO KEEP TRACK OF THE PROBABILITIES
C      ASSOCIATED WITH INDIVIDUAL DUAL TRIPS WHICH ARE
C      FORMED BY CHOOSING TWO OPENINGS RANDOMLY
C
      WEIT=0.0
      NSL=ICOL*NOL
      NN=10000
      IF(NSL.GE.10000)NN=15000
C
C      START SIMULATION IN ORDER TO FIND DUAL CYCLE MEAN
C      AND VARIANCE. NOTE THAT SIMULATION TIME= 'NN'
C
      DO 20 K20=1,NN
      JPR=(RANF(X)*NOL)+0.4999
      JP=(RANF(X)*ICOL)+0.4999
      KG=(RANF(X)*NOL)+0.4999
      KLM=(RANF(X)*ICOL)+0.4999
      SLT=(ABS((KLM-JP)*WTH))/HVEL
      VTTM=(ABS((KG-JPR)*HEHT))/VVEL
      IF(VTTM.GT.SLT) SLT=VTTM
      TTAYM=TIME(JPR,JP)+TIME(KG,KLM)+SLT
      MA1=IPRD(JPR,JP)
      MA2=IPRD(KG,KLM)
      IF(MA1*MA2)49,20,49
49    BOL=MIA(MA1)*MIA(MA2)*DELT
      PROB=(TNC(MA1)*TNC(MA2))/BOL
      RTAM=RTAM+(TTAYM*PROB)
      STAM=STAM+(TTAYM*TTAYM*PROB)
      WEIT=WEIT+PROB
20    CONTINUE
C
C      COMPUTE THE WEIGHTED AVERAGE OF THE TRIP TIMES
C      GENERATED THROUGH SIMULATION
C
      EDT=RTAM/WEIT
      VDT=(STAM/WEIT)-EDT**2
      RETURN
      END
```

APPENDIX 8

Outputs obtained from those programs listed

in Appendices 6 and 7

```
? 25.,25

    TRUE SINGLE(VAR) & DUAL(VAR) TIMES    .3783   .0353   .5224   .0303
      .158 CP SECONDS EXECUTION TIME
? 100.,20
? 10.,30

    TRUE SINGLE(VAR) & DUAL(VAR) TIMES    .2659   .0169   .3710   .0198
      .193 CP SECONDS EXECUTION TIME

? 50.,30
? 30.,30
? 30.,45

    TRUE SINGLE(VAR) & DUAL(VAR) TIMES    .5174   .0708   .7182   .0630
      .676 CP SECONDS EXECUTION TIME

? 90.,30
? 30.,30
? 15.,45

    TRUE SINGLE(VAR) & DUAL(VAR) TIMES    .4096   .0473   .5748   .0507
      .626 CP SECONDS EXECUTION TIME
/LGO
? 30.,30
? 30.,30
? 45.,45

    TRUE SINGLE(VAR) & DUAL(VAR) TIMES    .6131   .0751   .8239   .0537
      .704 CP SECONDS EXECUTION TIME
? 100.,50
? 90.,50
? 80.,50
? 70.,50

    TRUE SINGLE(VAR) & DUAL(VAR) TIMES    .8172   .1550  1.1112   .1204
     2.818 CP SECONDS EXECUTION TIME
? 100.,50
? 85.,50
? 84.,50
? 70.,50

    TRUE SINGLE(VAR) & DUAL(VAR) TIMES    .8216   .1557  1.1157   .1190
     2.184 CP SECONDS EXECUTION TIME
```

```
? 100.,25
? 70.,35
? 60.,60
? 40.,80


    TRUE SINGLE(VAR) & DUAL(VAR) TIMES    .5922   .1306   .8490   .1366
     3.281 CP SECONDS EXECUTION TIME

? 100.,50
? 98.,50
? 96.,50
? 94.,50


    TRUE SINGLE(VAR) & DUAL(VAR) TIMES    .8586   .1589  1.1554   .1153
     3.023 CP SECONDS EXECUTION TIME

? 300.,60
? 240.,60
? 180.,60
? 120.,60
? 60.,60


    TRUE SINGLE(VAR) & DUAL(VAR) TIMES    .7803   .1120  1.0742   .0998
     4.744 CP SECONDS EXECUTION TIME

? 200.,40
? 180.,60
? 150.,80
? 110.,100
? 60.,120


    TRUE SINGLE(VAR) & DUAL(VAR) TIMES    .7775   .1413  1.0933   .1325
     8.803 CP SECONDS EXECUTION TIME

? 164.,80
? 160.,80
? 156.,80
? 15.0 *DEL*
152.,80
? 148.,80


    TRUE SINGLE(VAR) & DUAL(VAR) TIMES   1.0423   .1403  1.4101   .1116
    13.948 CP SECONDS EXECUTION TIME
```

```
? 50.,25
? 25.,25

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .3783   .0353   .4646   .0226
      .067 CP SECONDS EXECUTION TIME

? 100.,20
? 10.,30

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .2659   .0169   .3465   .0140
      .064 CP SECONDS EXECUTION TIME

? 50.,30
? 30.,30
? 30.,45

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .5174   .0708   .6932   .0487
      .192 CP SECONDS EXECUTION TIME

? 90.,30
? 30.,30
? 15.,45

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .4096   .0473   .5852   .0373
      .202 CP SECONDS EXECUTION TIME

/LGO
? 30.,30
? 30.,30
? 45.,45

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .6131   .0751   .7788   .0486
      .193 CP SECONDS EXECUTION TIME

? 100.,50
? 90.,50
? 80.,50
? 70.,50

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .8172   .1550  1.0473   .1087
      .639 CP SECONDS EXECUTION TIME
```

```
? 100.,50
? 85.,50
? 84.,50
? 70.,50


    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .8216    .1557  1.0517    .1074
      .629 CP SECONDS EXECUTION TIME

? 100.,25
? 70.,35
? 60.,60
? 40.,80


    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .5922    .1306   .8076    .1145
      .641 CP SECONDS EXECUTION TIME
? 100.,50
? 98.,50
? 96.,50
? 94.,50


    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .8586    .1589  1.0887    .1069
      .655 CP SECONDS EXECUTION TIME
? 300.,60
? 240.,60
? 180.,60
? 120.,60
? 60.,60


    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .7303    .1120  1.0466    .0840
     1.250 CP SECONDS EXECUTION TIME
? 200.,40
? 180.,60
? 150.,80
? 110.,100
? 60.,120


    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .7775    .1413  1.0800    .1230
     2.205 CP SECONDS EXECUTION TIME
? 164.,80
? 160.,80
? 156.,80
? 152.,80
? 148.,80


    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES   1.0423    .1403  1.3614    .1051
     2.242 CP SECONDS EXECUTION TIME
```

```
? 50.,25
? 25.,205 *DEL*
25.,25

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .3783   .0353   .4797   .0262
     1.660 CP SECONDS EXECUTION TIME

? 100.,20
? 10.,30

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .2659   .0169   .3466   .0154
     1.662 CP SECONDS EXECUTION TIME

? 50.,30
? 30.,30
? 30.,45

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .5174   .0708   .6767   .0558
     1.519 CP SECONDS EXECUTION TIME

? 90.,30
? 30.,30
? 15.,45

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .4096   .0473   .5481   .0433
     1.819 CP SECONDS EXECUTION TIME

? 30.,30
? 30.,30
? 45.,45

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .6131   .0751   .7775   .0496
     1.603 CP SECONDS EXECUTION TIME

? 100.,50
? 90.,50
? 80.,50
? 70.,50

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .8172   .1550  1.0662   .1112
     2.074 CP SECONDS EXECUTION TIME
```

```
? 100.,50
? 85.,50
? 84.,50
? 70.,50

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .8216    .1557  1.0711    .1101
     2.045 CP SECONDS EXECUTION TIME

? 100.,25
? 70.,35
? 60.,60
? 40.,80

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .5922    .1306   .8121    .1229
     1.950 CP SECONDS EXECUTION TIME

? 100.,50
? 98.,50
? 96.,50
? 94.,50

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .8586    .1589  1.1079    .1073
     1.846 CP SECONDS EXECUTION TIME
? 300.,60
? 240.,60
? 180.,60
? 120.,60
? 60.,60

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .7803    .1120  1.0506    .0918
     3.276 CP SECONDS EXECUTION TIME
? 200.,40
? 180.,60
? 150.,80
? 110.,100
? 60.,120

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES    .7775    .1413  1.0680    .1281
     3.387 CP SECONDS EXECUTION TIME
? 164.,80
? 160.,80
? 156.,80
? 152.,80
? 148.,80

    SIMULATED SINGLE(VAR) & DUAL(VAR) TIMES  1.0123    .1493  1.3707    .1075
     4.308 CP SECONDS EXECUTION TIME
```

APPENDIX 9

Sample runs to demonstrate that the expected mean and variance of travel time under dedicated storage are minimized when b is maximized

THE RACK IS SQUARE IN TIME FOR 2 LEVELS

NUMBER OF LEVELS= 1

SINGLE COMMAND(MEAN & VAR.) .3535 .0720 DUAL COMMAND(MEAN & VAR.) .5147 .0934

NUMBER OF LEVELS= 2

SINGLE COMMAND(MEAN & VAR.) .2949 .0207 DUAL COMMAND(MEAN & VAR.) .4118 .0236

NUMBER OF LEVELS= 3

SINGLE COMMAND(MEAN & VAR.) .3171 .0373 DUAL COMMAND(MEAN & VAR.) .4497 .0451

NUMBER OF LEVELS= 4

SINGLE COMMAND(MEAN & VAR.) .3583 .0722 DUAL COMMAND(MEAN & VAR.) .5196 .0910

NUMBER OF LEVELS= 5

SINGLE COMMAND(MEAN & VAR.) .4267 .1273 DUAL COMMAND(MEAN & VAR.) .6311 .1560

THE RACK IS SQUARE IN TIME FOR 3 LEVELS

NUMBER OF LEVELS= 1

SINGLE COMMAND(MEAN & VAR.) .5159 .2067 DUAL COMMAND(MEAN & VAR.) .7704 .2471

NUMBER OF LEVELS= 2

SINGLE COMMAND(MEAN & VAR.) .3146 .0398 DUAL COMMAND(MEAN & VAR.) .4562 .0490

NUMBER OF LEVELS= 3

SINGLE COMMAND(MEAN & VAR.) .2875 .0189 DUAL COMMAND(MEAN & VAR.) .4060 .0192

NUMBER OF LEVELS= 4

SINGLE COMMAND(MEAN & VAR.) .2912 .0214 DUAL COMMAND(MEAN & VAR.) .4126 .0225

NUMBER OF LEVELS= 5

SINGLE COMMAND(MEAN & VAR.) .3076 .0332 DUAL COMMAND(MEAN & VAR.) .4404 .0379

THE RACK IS SQUARE IN TIME FOR 4 LEVELS

NUMBER OF LEVELS= 2

SINGLE COMMAND(MEAN & VAR.)   .1750 .0159   DUAL COMMAND(MEAN & VAR.)   .2547 .0167

NUMBER OF LEVELS= 3

SINGLE COMMAND(MEAN & VAR.)   .1372 .0056   DUAL COMMAND(MEAN & VAR.)   .1951 .0057

NUMBER OF LEVELS= 4

SINGLE COMMAND(MEAN & VAR.)   .1283 .0033   DUAL COMMAND(MEAN & VAR.)   .1804 .0031

NUMBER OF LEVELS= 5

SINGLE COMMAND(MEAN & VAR.)   .1295 .0036   DUAL COMMAND(MEAN & VAR.)   .1823 .0034

NUMBER OF LEVELS= 6

SINGLE COMMAND(MEAN & VAR.)   .1328 .0045   DUAL COMMAND(MEAN & VAR.)   .1881 .0045

THE RACK IS SQUARE IN TIME FOR 4 LEVELS

NUMBER OF LEVELS= 2

SINGLE COMMAND(MEAN & VAR.) .2411 .0289 DUAL COMMAND(MEAN & VAR.) .3505 .0305

NUMBER OF LEVELS= 3

SINGLE COMMAND(MEAN & VAR.) .1922 .0102 DUAL COMMAND(MEAN & VAR.) .2712 .0100

NUMBER OF LEVELS= 4

SINGLE COMMAND(MEAN & VAR.) .1825 .0066 DUAL COMMAND(MEAN & VAR.) .2556 .0060

NUMBER OF LEVELS= 5

SINGLE COMMAND(MEAN & VAR.) .1861 .0079 DUAL COMMAND(MEAN & VAR.) .2626 .0078

NUMBER OF LEVELS= 6

SINGLE COMMAND(MEAN & VAR.) .1924 .0102 DUAL COMMAND(MEAN & VAR.) .2735 .0106

THE RACK IS SQUARE IN TIME FOR 5 LEVELS

NUMBER OF LEVELS= 3

SINGLE COMMAND(MEAN & VAR.) .3569 .0321 DUAL COMMAND(MEAN & VAR.) .4803 .0226

NUMBER OF LEVELS= 4

SINGLE COMMAND(MEAN & VAR.) .2951 .0140 DUAL COMMAND(MEAN & VAR.) .3988 .0106

NUMBER OF LEVELS= 5

SINGLE COMMAND(MEAN & VAR.) .2814 .0100 DUAL COMMAND(MEAN & VAR.) .3811 .0078

NUMBER OF LEVELS= 6

SINGLE COMMAND(MEAN & VAR.) .2909 .0128 DUAL COMMAND(MEAN & VAR.) .3935 .0098

NUMBER OF LEVELS= 7

SINGLE COMMAND(MEAN & VAR.) .3232 .0220 DUAL COMMAND(MEAN & VAR.) .4357 .0158

THE RACK IS SQUARE IN TIME FOR 6 LEVELS

NUMBER OF LEVELS= 4

SINGLE COMMAND(MEAN & VAR.) .8750 .2577 DUAL COMMAND(MEAN & VAR.) 1.2300 .2442

NUMBER OF LEVELS= 5

SINGLE COMMAND(MEAN & VAR.) .7875 .1456 DUAL COMMAND(MEAN & VAR.) 1.0984 .1357

NUMBER OF LEVELS= 6

SINGLE COMMAND(MEAN & VAR.) .7595 .1107 DUAL COMMAND(MEAN & VAR.) 1.0544 .0992

NUMBER OF LEVELS= 7

SINGLE COMMAND(MEAN & VAR.) .7687 .1219 DUAL COMMAND(MEAN & VAR.) 1.0679 .1097

NUMBER OF LEVELS= 8

SINGLE COMMAND(MEAN & VAR.) .8026 .1640 DUAL COMMAND(MEAN & VAR.) 1.1210 .1531

THE RACK IS SQUARE IN TIME FOR 7 LEVELS

NUMBER OF LEVELS= 5

SINGLE COMMAND(MEAN & VAR.) .2187 .0154 DUAL COMMAND(MEAN & VAR.) .3141 .0178

NUMBER OF LEVELS= 6

SINGLE COMMAND(MEAN & VAR.) .2110 .0111 DUAL COMMAND(MEAN & VAR.) .2998 .0117

NUMBER OF LEVELS= 7

SINGLE COMMAND(MEAN & VAR.) .2088 .0100 DUAL COMMAND(MEAN & VAR.) .2958 .0101

NUMBER OF LEVELS= 8

SINGLE COMMAND(MEAN & VAR.) .2090 .0101 DUAL COMMAND(MEAN & VAR.) .2963 .0102

NUMBER OF LEVELS= 9

SINGLE COMMAND(MEAN & VAR.) .2119 .0116 DUAL COMMAND(MEAN & VAR.) .3013 .0123

THE RACK IS SQUARE IN TIME FOR 7 LEVELS

NUMBER OF LEVELS= 5

SINGLE COMMAND(MEAN & VAR.) .2202 .0156 DUAL COMMAND(MEAN & VAR.) .3163 .0180

NUMBER OF LEVELS= 6

SINGLE COMMAND(MEAN & VAR.) .2123 .0113 DUAL COMMAND(MEAN & VAR.) .3016 .0118

NUMBER OF LEVELS= 7

SINGLE COMMAND(MEAN & VAR.) .2101 .0101 DUAL COMMAND(MEAN & VAR.) .2976 .0102

NUMBER OF LEVELS= 8

SINGLE COMMAND(MEAN & VAR.) .2103 .0102 DUAL COMMAND(MEAN & VAR.) .2980 .0103

NUMBER OF LEVELS= 9

SINGLE COMMAND(MEAN & VAR.) .2132 .0117 DUAL COMMAND(MEAN & VAR.) .3032 .0124

THE RACK IS SQUARE IN TIME FOR 8 LEVELS

NUMBER OF LEVELS= 6

SINGLE COMMAND(MEAN & VAR.) .2861 .0293 DUAL COMMAND(MEAN & VAR.) .4130 .0338

NUMBER OF LEVELS= 7

SINGLE COMMAND(MEAN & VAR.) .2792 .0238 DUAL COMMAND(MEAN & VAR.) .4007 .0262

NUMBER OF LEVELS= 8

SINGLE COMMAND(MEAN & VAR.) .2786 .0233 DUAL COMMAND(MEAN & VAR.) .3997 .0255

NUMBER OF LEVELS= 9

SINGLE COMMAND(MEAN & VAR.) .2820 .0260 DUAL COMMAND(MEAN & VAR.) .4059 .0293

NUMBER OF LEVELS= 10

SINGLE COMMAND(MEAN & VAR.) .2854 .0288 DUAL COMMAND(MEAN & VAR.) .4120 .0332

APPENDIX 10

Program listing from which sample runs presented

in Appendix 9 were obtained

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C       FILE NAME: TEMP
C       THIS PROGRAM WAS WRITTEN WITH THE PURPOSE OF
C       SHOWING THAT UNDER DEDICATED STORAGE THE EXPECTED
C       MEAN AND VARIANCE OF DUAL CYCLES WILL BE MINIMIZED
C       AT B=1.
C
C
      DIMENSION IA(10),C(10),MIA(10),TNC(10)
      COMMON/BLK1/HEHT,WTH,HVEL,VVEL,CAPC,DELT,TNC,C,RR,
     *NOL,ICOL,NOPR,MIA
C
C       INITIALIZE SYSTEM PARAMETERS
C       NOPR=NUMBER OF PRODUCT CLASSES
C
      HEHT=42.0
      WTH=44.0
      HVEL=220.0
      VVEL=80.0
      NOAI=1
C
C       'NOAI' HAS NOT BEEN EXCLUDED IN ORDER TO
C       FACILITATE POSSIBLE MODIFICATIONS OF THE PROGRAM
C
      NOPR=3
C
C       INITIALIZE THE (C/A) VALUES FOR EACH PRODUCT CLASS
C
      IA(3)=150
      IA(2)=100
      IA(1)=50
      C(3)=1.0
      C(2)=2.0
      C(1)=5.0
      HEHT=HEHT/12.0
      WTH=WTH/12.0
      RAT=HVEL/VVEL
      TA=0.0
      CAPC=0.0
      DELT=0.0
      DO 2 K1=1,NOPR
      TA=TA+IA(K1)
2     CAPC=CAPC+C(K1)
      PA=TA/(NOAI*2)
      ICM=0
      DO 3 K2=1,NOPR
      MIA(K2)=((IA(K2)/TA)*PA)+1.0
      ICM=ICM+MIA(K2)
3     TNC(K2)=C(K2)/CAPC
      DO 4 K3=1,NOPR
4     DELT=DELT+((TNC(K3)**2/MIA(K3)**2)*MIA(K3))
      DELT=(1.0-DELT)/2.0
C
C       INITIALLY INOL IS THE NUMBER OF LEVELS THAT
C       MAXIMIZE B.
C
      INOL=((SQRT((ICM*WTH*HEHT)/RAT))/HEHT)+0.5
      WRITE(6,14)INOL
14    FORMAT(///3X,'THE RACK IS SQUARE IN TIME FOR',1X,I2,1X,'LEVELS
     *'/)
      INOL=INOL-2
      IF(INOL.LE.0)INOL=1
      JNOL=INOL+4
C
C       START ITERATING OVER THE RANGE INOL-2
C       TO INOL+2.
C
      DO 5 NOL=INOL,JNOL
      ICOL=(PA/NOL)+1.0
      NOAS=ICOL*NOL
      RR=(ICOL*WTH)/(NOL*HEHT)
      CALL TRAVEL(ETS,VTS,EDT,VDT)
      WRITE(6,8)NOL
```

```
8     FORMAT(/3X,'NUMBER OF LEVELS=',1X,I2)
      WRITE(6,7)ETS,VTS,EDT,VDT
7     FORMAT(/3X,'SINGLE COMMAND(MEAN & VAR.)',1X,2F7.4,3X,'DUAL COMMAND
     *(MEAN & VAR.)',1X,2F7.4/)
5     CONTINUE
      STOP
      END

      SUBROUTINE TRAVEL(ETS,VTS,EDT,VDT)
C
C      THIS SUBROUTINE DETERMINES THE EXPECTED MEAN AND
C      VARIANCE OF SINGLE AND DUAL CYCLES BY USING
C      THE COMPLETE ENUMERATION SCHEME
C
      DIMENSION TIME(50,300),IPRD(50,300),MIA(10),C(10),TNC(10)
      DIMENSION SC(10),RH(10),RV(10),ASC(10),RFC(10),RM(10),DT(10)
      COMMON/BLK1/HEHT,WTH,HVEL,VVEL,CAPC,DELT,TNC,C,RR,
     *NOL,ICOL,NOPR,MIA
      TOTM=0.0
      TOTS=0.0
C
C      FIND THE TRAVEL TIME TO EACH OPENING
C      AND STORE IN 'TIME'.
C      CUTM - STORES SINGLE CYCLE TRAVEL TIME CUMULATIVELY
C      CUTS - STORES THE SQUARE OF SINGLE CYCLE TRAVEL
C             TIME CUMULATIVELY
C      ETS - EXPECTED MEAN TRAVEL TIME OF SINGLE CYCLE
C      VTS - EXPECTED VARIANCE OF SINGLE CYCLE
C      STAM - STORES THE ADJUSTED DUAL CYCLE TRAVEL
C             TIME CUMULATIVELY
C      RTAM - STORES THE SQUARE OF THE ADJUSTED DUAL
C             CYCLE TRAVEL TIME CUMULATIVELY
C      EDT - EXPECTED MEAN TRAVEL TIME OF DUAL CYCLE
C      VDT - EXPECTED VARIANCE OF DUAL CYCLE
C
      PHET=-HEHT/2.0
      DO 12 JM=1,NOL
      PHET=PHET+HEHT
      FWT=WTH/2.0
      DO 12 JC=1,ICOL
      TH=FWT/HVEL
      TV=PHET/VVEL
      TIME(JM,JC)=TH
      IF(TV.GT.TH)TIME(JM,JC)=TV
      FWT=FWT+WTH
12    IPRD(JM,JC)=0
C
C      ALLOCATE THE PRODUCT CLASSES TO THE RACK
C      OPENINGS STARTING FROM THE FIRST PRODUCT CLASS.
C      WHILE DOING THIS ALSO STORE THE TRAVEL
C      TIME AND ITS SQUARE IN 'CUTM' AND 'CUTS',RESPECTIVELY
C
      DO 16 NP=1,NOPR
      CUTM=0.0
      CUTS=0.0
      ILG=MIA(NP)
      DO 19 MN=1,ILG
      THINI=10000.0
      LX=1
      LY=1
      DO 17 ILE=1,NOL
      DO 17 ICO=1,ICOL
      IF(IPRD(ILE,ICO).NE.0) GO TO 17
      IF(TIME(ILE,ICO).GE.THINI) GO TO 17
      THINI=TIME(ILE,ICO)
      LX=ILE
      LY=ICO
17    CONTINUE
      IPRD(LX,LY)=NP
      CUTM=CUTM+TIME(LX,LY)
      CUTS=CUTS+(TIME(LX,LY)*TIME(LX,LY))
19    CONTINUE
```

```
      TOTM=TOTM+((C(NP)/ILG)*CUTM)
      TOTS=TOTS+((C(NP)/ILG)*CUTS)
16    CONTINUE
      ETS=TOTM*(2.0/CAPC)
      VTS=((4.0/CAPC)*TOTS)-((4/(CAPC*CAPC))*TOTM*TOTM)
C
C      COMPUTE EXPECTED MEAN AND VARIANCE OF DUAL
C       CYCLE TRAVEL TIME
C
      STAM=0.0
      RTAM=0.0
      NSL=ICOL*NOL
      NS1=NSL-1
      DO 88 L6=1,NS1
      JPR=((L6-0.00001)/ICOL)+1.0
      JP=L6-((JPR-1)*ICOL)
      L7=L6+1
      DO 88 L8=L7,NSL
      KG=((L8-0.00001)/ICOL)+1.0
      KLM=L8-((KG-1)*ICOL)
      SLT=(ABS((KLM-JP)*WTH))/HVEL
      VTTM=(ABS((KG-JPR)*HEHT))/VVEL
      IF(VTTM.GT.SLT) SLT=VTTM
      TTAYM=TIME(JPR,JP)+TIME(KG,KLM)+SLT
      MA1=IPRD(JPR,JP)
      MA2=IPRD(KG,KLM)
      IF(MA1*MA2)49,88,49
49    BOL=MIA(MA1)*MIA(MA2)*DELT
      RTAM=RTAM+((TTAYM*TNC(MA1)*TNC(MA2))/BOL)
      STAM=STAM+((TTAYM*TTAYM*TNC(MA1)*TNC(MA2))/BOL)
88    CONTINUE
      EDT=RTAM
      VDT=STAM-(EDT*EDT)
      RETURN
      END
```

APPENDIX 11

Program listing of the algorithm developed to find an optimum design

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION LFIB(20),ANS(30),C(10),IA(10),IBPT(4),FACT(10)
      COMMON ANOP,WTH,HEHT,DEPTH,WTHP,HEHTP,DEPTHP,DUAL,ALAMDA,
     *VVEL,HVEL,RATIO,OPERN1,OPERN2,BCOST,RLCOST,SCOST1,SCOST2,RCOST,
     *C,ATMAR,RITR,FACT,PGIVA,RLBCOST,SRMCOST,BMCOST,
     *IPULL,IADD,NOPR,IA,IBPT,ISTM,IBLDG,ITPUT,INFES,IRACK
C
C     INPUT OF DATA
C
      WRITE(6,60)
60    FORMAT(//3X,'GEORGIA TECH. - DESIGN OF AN AS/RS')
      WRITE(6,61)
61    FORMAT(//3X,'TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)')
      READ(5,*)DEPTH,WTH,HEHT
      WTHP=(WTH+8.0)/12.0
      HEHTP=(HEHT+10.0)/12.0
      DEPTHP=(DEPTH+6.0)/12.0
      WRITE(6,62)
62    FORMAT(3X,'TYPE UNIT LOAD WEIGHT(LB.S)')
      READ(5,*)WEGT
      WRITE(6,63)
63    FORMAT(3X,'TYPE DUAL COMMAND PERCENTAGE')
      READ(5,*)DUAL
      WRITE(6,64)
64    FORMAT(3X,'TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)')
      READ(5,*)VVEL,HVEL
      RATIO=HVEL/VVEL
      WRITE(6,65)
65    FORMAT(3X,'TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)')
      READ(5,*)OPERN1,OPERN2
      WRITE(6,66)
66    FORMAT(3X,'TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)')
      READ(5,*)BCOST,RLCOST
      WRITE(6,67)
67    FORMAT(3X,'TYPE 0,1,2 OR 3 FOR S/R LOGIC')
      WRITE(6,505)
505   FORMAT(5X,'0=MAN-ON-BOARD  1=ON THE S/R')
      WRITE(6,506)
506   FORMAT(5X,'2=OFF THE S/R  3=CENTRAL CONSOLE')
      READ(5,*)LOGC
      RLBCOST=0.0
      IF(LOGC.NE.0)GO TO 501
      WRITE(6,502)
502   FORMAT(3X,'TYPE ANNUAL COST/OPERATOR')
      READ(5,*)RLBCOST
501   WRITE(6,503)
503   FORMAT(3X,'TYPE ANNUAL MAINTENANCE COST/SR')
      READ(5,*)SRMCOST
      WRITE(6,504)
504   FORMAT(3X,'TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.')
      READ(5,*)BMCOST
      WRITE(6,68)
68    FORMAT(3X,'TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.')
      READ(5,*)IBLDG
      WRITE(6,69)
69    FORMAT(3X,'TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1')
      READ(5,*)ITPUT
      WRITE(6,76)
76    FORMAT(3X,'TYPE ATMARR,APPLICABLE INCOME TAX RATE')
      READ(5,*)ATMAR,RITR
      WRITE(6,70)
70    FORMAT(3X,'TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE')
      READ(5,*)ISTM
      IF(ISTM.EQ.0)GO TO 2
      WRITE(6,71)
71    FORMAT(3X,'TYPE NO. OF PRODUCT CLASSES(INTEGER)')
      READ(5,*)NOPR
      WRITE(6,72)
72    FORMAT(3X,'TYPE OP.N/HR,NO. OF SLOTS FOR EACH PROD. CLASS')
      ALAMDA=0.0
      ANOP=0.0
      DO 3 K1=1,NOPR
      READ(5,*)C(K1),IA(K1)
      ALAMDA=ALAMDA+C(K1)
3     CONTINUE
```

```
      WRITE(6,102)
102   FORMAT(3X,'FOR ENUMERATION TYPE 1,FOR SIMULATION TYPE 0')
      READ(5,*)IFULL
      GO TO 4
2     WRITE(6,73)
73    FORMAT(3X,'TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)')
      READ(5,*)ANOP
      WRITE(6,74)
74    FORMAT(3X,'TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)')
      READ(5,*)ALAMDA
C
C     END-OF-DATA-INPUT
C     COMPUTE CONSTANT COSTS AND PARAMETERS
C
4     IBPT(1)=(30.0/HEHTP)+0.000001
      IBPT(2)=(42.0/HEHTP)+0.000001
      IBPT(3)=(60.0/HEHTP)+0.000001
      IBPT(4)=(85.0/HEHTP)+0.000001
      PGIVA=((1.0+ATMAR)**10-1.0)/(ATMAR*((1.0+ATMAR)**10))
      DO 21 K21=1,10
21    FACT(K21)=1.0/((1.0+ATMAR)**K21)
      IADD=12.504243+(0.447478*WTH)+1.0
      ALS=(WTH*HEHT*DEPTH)/1728.0
      RCOST=1.579451+(0.025*ALS)+(0.0004424*WEGT)
     $-((WEGT*WEGT)/82500000.0)
      IF(WEGT.LT.1000.0) SCOST1=13000.0
      IF(WEGT.GE.1000.0.AND.WEGT.LT.3500.0) SCOST1=26000.0
      IF(WEGT.GE.3500.0.AND.WEGT.LT.6500.0) SCOST1=39000.0
      IF(WEGT.GE.6500.0) SCOST1=52000.0
      IF(LOGC.EQ.0)SCOST2=13000.0
      IF(LOGC.EQ.1)SCOST2=26000.0
      IF(LOGC.EQ.2)SCOST2=39000.0
      IF(LOGC.EQ.3)SCOST2=52000.0
      IRACK=0
      NFE=4
      LFIB(1)=0
      LFIB(2)=1
      DO 5 K=3,20
5     LFIB(K)=LFIB(K-1)+LFIB(K-2)
      MLL=0
      MUL=40
      CALL DOLRS(MUL,SYSC,ANS)
      IF(INFES.EQ.1)GO TO 12
      KK=1
      MX1=(MLL)+((LFIB(NFE-KK+1)*(MUL-MLL))/LFIB(NFE-KK+3))
      CALL DOLRS(MX1,AK,ANS)
      MX2=(MLL)+((LFIB(NFE-KK+2)*(MUL-MLL))/LFIB(NFE-KK+3))
      CALL DOLRS(MX2,BK,ANS)
      DO 7 K6=2,2
      IF(AK.GT.BK)GO TO 6
      MUL=MX2
      MX2=MX1
      BK=AK
      MX1=(MLL)+((LFIB(NFE-K6+1)*(MUL-MLL))/LFIB(NFE-K6+3))
      CALL DOLRS(MX1,AK,ANS)
      GO TO 7
6     MLL=MX1
      MX1=MX2
      AK=BK
      MX2=(MLL)+((LFIB(NFE-K6+2)*(MUL-MLL))/LFIB(NFE-K6+3))
      CALL DOLRS(MX2,BK,ANS)
7     CONTINUE
      IF(AK.GT.BK)GO TO 8
      MUL=MX2
      MX2=MX1+1
      CALL DOLRS(MX2,BK,ANS)
      IF(AK.GT.BK)MLL=MX1
      IF(AK.LE.BK)MUL=MX2
      GO TO 9
8     MLL=MX1
      AK=BK
      MX2=MX2+1
      CALL DOLRS(MX2,BK,ANS)
      IF(AK.GT.BK)MLL=MX2-1
      IF(AK.LE.BK)MUL=MX2
9     IF(MLL.EQ.0)MLL=1
```

```
C
C      END OF FIBONACCI SEARCH
C      FINAL INTERVAL OF UNCERTAINITY IS (MLL,MUL)
C
       JJ=0
       IRACK=1
       TAMIN=999999999.0
       DO 11 MM=MLL,MUL
       CALL DOLRS(MM,AMIN,ANS)
       IF(ANS(1).EQ.0.0)GO TO 11
       IF(AMIN.GT.TAMIN)JJ=1
       WRITE(6,10)MM
10     FORMAT(///3X,'NO. OF AISLES',1X,I4)
       WRITE(6,30)ANS(1)
30     FORMAT(3X,'NO. OF LEVELS',1X,F9.2)
       WRITE(6,31)ANS(2)
31     FORMAT(3X,'NO. OF COLUMNS',1X,F9.2)
       WRITE(6,32)ANS(3)
32     FORMAT(/3X,'BLDG. LENGTH',1X,F9.2)
       WRITE(6,33)ANS(4)
33     FORMAT(3X,'BLDG. WIDTH',1X,F9.2)
       WRITE(6,34)ANS(5)
34     FORMAT(3X,'BLDG. HEIGHT',1X,F9.2)
       WRITE(6,35)ANS(6)
35     FORMAT(/3X,'LAND COST',1X,F9.2)
       WRITE(6,36)ANS(7)
36     FORMAT(3X,'BLDG. COST',1X,F9.2)
       WRITE(6,37)ANS(8)
37     FORMAT(3X,'RACK COST',1X,F9.2)
       WRITE(6,38)ANS(9)
38     FORMAT(3X,'S/R MACHINE COST',1X,F9.2,3X,'DOLLARS')
       WRITE(6,600)
600    FORMAT(/3X,'THE RECURRING COSTS ARE'/)
       WRITE(6,601)ANS(16)
601    FORMAT(3X,'BLDG. MAINTENANCE COST',1X,F11.2)
       WRITE(6,602)ANS(17)
602    FORMAT(3X,'S/R MAINTENANCE COST',1X,F11.2)
       IF(LOGC.NE.0)GO TO 603
       WRITE(6,604)ANS(18)
604    FORMAT(3X,'TOTAL OPERATOR COST',1X,F11.2,1X,'DOLLARS/YR.')
603    WRITE(6,39)ANS(11)
39     FORMAT(/3X,'SYSTEM THROUGHPUT(0)',1X,F9.2,1X,'OP.NS/HR.')
       WRITE(6,40)ANS(12)
40     FORMAT(3X,'SYSTEM THROUGHPUT(1)',1X,F9.2,1X,'OP.NS/HR.')
       WRITE(6,41)ANS(13),ANS(14)
41     FORMAT(/3X,'EXP. SINGLE & DUAL COMMAND TRAVEL TIMES',1X,2F7.2)
       WRITE(6,42)ANS(15)
42     FORMAT(3X,'THE SHAPE FACTOR IS',1X,F9.2/)
       IF(ISTM.EQ.1)GO TO 13
       WRITE(6,43)ANS(19)
43     FORMAT(3X,'TOTAL NO. OF OPENINGS',1X,F9.2)
       GO TO 14
13     DO 15 K7=1,NOPR
       WRITE(6,44)ANS(18+K7),K7
44     FORMAT(3X,F9.2,1X,'OPENINGS FOR PROD. CLASS',1X,I3)
15     CONTINUE
14     WRITE(6,45)AMIN
45     FORMAT(//3X,'PW COST OF ABOVE DESIGN IS',1X,F12.2,1X,'DOLLARS')
       WRITE(6,48)ANS(10)
48     FORMAT(3X,'(TAX LIABILITY SHOULD BE >',1X,F9.2,')'///)
       IF(JJ.EQ.1)GO TO 16
11     TAMIN=AMIN
       GO TO 16
12     WRITE(6,46)
46     FORMAT(//3X,'WITH THE GIVEN INPUT 40 AISLES IS NOT ENOUGH TO MEET
      *THE REQUIRED THROUGHPUT,REVIEW YOUR INPUT DATA')
16     STOP
       END
       SUBROUTINE DOLRS(NOAI,AMIN,ANS)
C
C       THIS IS THE KEY SUBROUTINE USED IN MINIMIZING THE SYSTEM
C       COST WITHOUT VIOLATING THE THROUGHPUT REQUIREMENT. IT FINDS
C       THE CONSTRAINT-FREE OPTIMUM NUMBER OF LEVELS FOR THE
C       FIXED NUMBER OF AISLES TRANSFERRED FROM THE MAIN PROGRAM.
C       THEN IT WORKS TOWARDS A RACK THAT SATISFIES THE THROUGHPUT
C       IN CASE THE ABOVE FOUND OPTIMUM NUMBER OF LEVELS DO NOT
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```
C       MEET THE THROUGHPUT CONSTRAINT. AFTER FINDING THE
C       APPROPRIATE OPTIMUM NUMBER OF LEVELS, IT RETURNS THE
C       CORRESPONDING COST OF THE DESIGN TO THE MAIN PROGRAM.
C
       DIMENSION ANS(30),IA(10),C(10),MIA(10),AC(10),TNC(10),
      *IBPT(4),SOLN(30),FACT(10)
       COMMON ANOP,WTH,HEHT,DEPTH,WTHP,HEHTP,DEPTHP,DUAL,ALAMDA,
      *VVEL,HVEL,RATIO,OPERN1,OPERN2,BCOST,RLCOST,SCOST1,SCOST2,RCOST,
      *C,ATMAR,RITR,FACT,PGIVA,RLBCOST,SRMCOST,BMCOST,
      *IPULL,IADD,NOPR,IA,IBPT,ISTM,IBLDG,ITPUT,INFES,IRACK
       INFES=0
       NOA=(ANOP/(NOAI*2.0))+0.99999
       MNOL=IBPT(4)
       BLA=ALAMDA/NOAI
       IF(ISTM.EQ.0)GO TO 2
       CAPC=0.0
       NOA=0
       DO 3 K1=1,NOPR
       MIA(K1)=((IA(K1)*1.0)/(NOAI*2.0))+0.99999
       NOA=NOA+MIA(K1)
       AC(K1)=C(K1)/NOAI
3      CAPC=CAPC+AC(K1)
       IF(NOA.GE.400.AND.IPULL.EQ.0)GO TO 6
       DO 4 K2=1,NOPR
4      TNC(K2)=AC(K2)/CAPC
       DELT=0.0
       DO 24 K24=1,NOPR
24     DELT=DELT+((TNC(K24)**2/MIA(K24)**2)*MIA(K24))
       DELT=(1.0-DELT)/2.0
2      NOL=((SQRT((NOA*WTHP*HEHTP)/RATIO))/HEHTP)+0.5
       ICOL=((NOA*1.0)/NOL)+1.0
       IF(ISTM.EQ.0)GO TO 5
       CALL DTRAV(NOL,ICOL,COMP,OTHER,SC,DC,BB,
      *CAPC,DELT,AC,MIA,TNC)
       IF(COMP.LT.CAPC)GO TO 6
       GO TO 7
5      CALL RTRAV(NOL,ICOL,BLA,COMP,OTHER,SC,DC,BB)
       IF(COMP.LT.BLA)GO TO 6
7      BWTH=DEPTHP*3*NOAI
       IF(IBLDG.EQ.1)BWTH=BWTH+2.0
       GM1=NOA*WTHP*BWTH*BCOST
       GM2=HEHTP*IADD*BWTH*BCOST
       GM3=NOA*WTHP*BWTH*RLCOST
       GM4=NOA*WTHP*BWTH*BMCOST*PGIVA
       ALP1=GM3+(0.9694288*GM1)+GM4
       ALP1=GM3+(0.9694288*GM1)
       ALP2=(0.0002698*HEHTP**2*GM1)-(0.0031906*GM2)+(2.6582*NOA*NOAI)
       ALP3=0.0005396*HEHTP*GM2
       VAR=ALP2+ALP3
       IF(VAR.GT.ALP1)GO TO 8
       DO 9 K9=1,MNOL
       DIFR=(K9**2*ALP2)+(K9*K9*K9*ALP3)-ALP1
       IF(DIFR.GE.0.0)GO TO 10
9      CONTINUE
       IONOL=MNOL
       GO TO 11
10     IONOL=K9
       TCK9=(ALP1/IONOL)+(IONOL*ALP2)+(IONOL**2*(ALP3/2.0))
       K8=IONOL-1
       TCK8=(ALP1/K8)+(K8*ALP2)+(K8**2*(ALP3/2.0))
       IF(TCK8.LT.TCK9)IONOL=K8
       GO TO 11
8      IONOL=1
11     CALL COSTT(NOAI,IONOL,NOA,TKOST,SOLN)
       DO 13 K13=1,30
13     ANS(K13)=SOLN(K13)
       AMIN=TKOST
       LEVEL=IONOL
       IF(IONOL.LT.IBPT(1))GO TO 12
       DO 14 K14=1,4
       IF(IBPT(K14).GE.IONOL)GO TO 12
       CALL COSTT(NOAI,IBPT(K14),NOA,TKOST,SOLN)
       IF(TKOST.GE.AMIN)GO TO 14
       DO 23 K23=1,30
23     ANS(K23)=SOLN(K23)
```

```
      AMIN=TKOST
      LEVEL=IBPT(K14)
14    CONTINUE
12    RRR=BLA
      IF(ISTM.EQ.1)RRR=CAPC
      JCOL=ANS(2)
      IF(ISTM.EQ.0)CALL RTRAV(LEVEL,JCOL,BLA,COMP,OTHER,SC,DC,BB)
      IF(ISTM.EQ.1)CALL DTRAV(LEVEL,JCOL,COMP,OTHER,SC,DC,BB,
     *CAPC,DELT,AC,MIA,TNC)
      IF(COMP.GE.RRR)GO TO 99
      IF(LEVEL.GT.NOL)GO TO 17
      LEV2=LEVEL+1
      DO 18 K18=LEV2,MNOL
      ICOL=((NOA*1.0)/K18)+0.99999
      IF(ISTM.EQ.0)CALL RTRAV(K18,ICOL,BLA,COMP,OTHER,SC,DC,BB)
      IF(ISTM.EQ.1)CALL DTRAV(K18,ICOL,COMP,OTHER,SC,DC,BB,
     *CAPC,DELT,AC,MIA,TNC)
      IF(COMP.GE.RRR)GO TO 19
18    CONTINUE
19    IFNOL=K18
      CALL COSTT(NOAI,IFNOL,NOA,TKOST,ANS)
      AMIN=TKOST
      GO TO 99
17    DO 20 K20=1,LEVEL
      II=LEVEL-K20
      ICOL=((NOA*1.0)/II)+0.99999
      IF(ISTM.EQ.0)CALL RTRAV(II,ICOL,BLA,COMP,OTHER,SC,DC,BB)
      IF(ISTM.EQ.1)CALL DTRAV(II,ICOL,COMP,OTHER,SC,DC,BB,
     *CAPC,DELT,AC,MIA,TNC)
      IF(COMP.GE.RRR)GO TO 21
20    CONTINUE
21    IFNOL=II
      CALL COSTT(NOAI,IFNOL,NOA,TKOST,ANS)
      AMIN=TKOST
      GO TO 99
6     INFES=1
      DO 94 K94=1,30
94    ANS(K94)=0.0
      AMIN=999999999.0
      GO TO 98
99    ANS(11)=COMP*NOAI
      ANS(12)=OTHER*NOAI
      IF(ANS(12).GT.ANS(11))GO TO 97
      ANS(11)=OTHER*NOAI
      ANS(12)=COMP*NOAI
97    ANS(13)=SC
      ANS(14)=DC
      ANS(15)=BB
      IF(ISTM.EQ.1)GO TO 96
      ANS(19)=ANS(1)*ANS(2)*2*NOAI
      GO TO 98
96    DO 95 K95=1,NOPR
95    ANS(18+K95)=MIA(K95)
98    RETURN
      END
      SUBROUTINE COSTT(NOAI,NOLL,NOA,TKOST,SOLN)
C
C       GIVEN THE DESIGN VARIABLES,THIS SUBROUTINE WILL RETURN
C       THE VALUES OF EACH VARIABLE COST ELEMENTS. IT USES
C       SUBROUTINE <PWORTH> TO FIND THE PRESENT WORTH OF
C       ABOVE COMPUTED VARIABLE COST ELEMENTS.
C
      DIMENSION SOLN(30),IA(10),C(10),FACT(10),IBPT(4)
      COMMON ANOP,WTH,HEHT,DEPTH,WTHP,HEHTP,DEPTHP,DUAL,ALAMDA,
     *VVEL,HVEL,RATIO,OPERN1,OPERN2,BCOST,RLCOST,SCOST1,SCOST2,RCOST,
     *C,ATMAR,RITR,FACT,PGIVA,RLBCOST,SRMCOST,BMCOST,
     *IPULL,IADD,NOPR,IA,IBPT,ISTM,IBLDG,ITPUT,INFES,IRACK
      DO 2 K2=1,30
2     SOLN(K2)=0.0
      ICOL=((NOA*1.0)/NOLL)+0.99999
      SOLN(1)=NOLL
      SOLN(2)=ICOL
      SOLN(3)=(WTHP*ICOL)+IADD
      SOLN(4)=(DEPTHP*3.0)*NOAI
      IF(IBLDG.EQ.1)SOLN(4)=SOLN(4)+2.0
```

```
      SOLN(5)=(HEHTP*NOLL)+4.0
      SOLN(6)=SOLN(3)*SOLN(4)*RLCOST
      SOLN(16)=SOLN(3)*SOLN(4)*BMCOST
      SOLN(17)=NOAI*SRMCOST
      SOLN(18)=NOAI*RLBCOST
      RECUR=SOLN(16)+SOLN(17)+SOLN(18)
      CFR=0.986508-(0.005349*SOLN(5))+(0.0002698*SOLN(5)**2)
      IF(SOLN(5).LE.25.0)CFR=1.0
      SOLN(7)=SOLN(3)*SOLN(4)*BCOST*CFR
      IF(IBLDG.EQ.0)SOLN(7)=SOLN(3)*SOLN(4)*((BCOST*CFR)-6.0)
      SOLN(8)=(RCOST+(0.0949367*NOLL))*SOLN(1)*SOLN(2)*2*NOAI*14.0
      IF(IRACK.EQ.1)SOLN(8)=(RCOST+((0.23328*NOLL)-(0.00476*
     *NOL**2)-0.654611))*SOLN(1)*SOLN(2)*NOAI*2*14.0
      TSR=SCOST1+SCOST2
      IF(NOLL.LE.IBPT(1))SOLN(9)=TSR+13000.0
      IF(NOLL.GT.IBPT(1).AND.NOLL.LE.IBPT(2))SOLN(9)=
     *TSR+26000.0
      IF(NOLL.GT.IBPT(2).AND.NOLL.LE.IBPT(3))SOLN(9)=
     *TSR+39000.0
      IF(NOLL.GT.IBPT(3).AND.NOLL.LE.IBPT(4))SOLN(9)=
     *TSR+52000.0
      IF(NOLL.GT.IBPT(4))SOLN(9)=999999999.0
      SOLN(9)=SOLN(9)*NOAI
      CALL PWORTH(NOAI,SOLN(6),SOLN(7),SOLN(8),SOLN(9),RECUR,YY,PWC)
      SOLN(10)=YY
      TKOST=PWC
      RETURN
      END
      SUBROUTINE RTRAV(NOL1,ICOL1,FLOW,COMP,OTHER,ST,DT,BB)
C
C      GIVEN THE RACK SHAPE AND TRAVEL VELOCITIES, THIS
C      SUBROUTINE RETURNS THE MEAN AND VARIANCE OF SINGLE
C      AND DUAL COMMAND TRAVEL TIMES UNDER 'RANDOMIZED'
C      STORAGE. IT ALSO RETURNS THE OVER-ALL MEAN AND
C      VARIANCE OF S/R TRAVEL TIME AND THE VALUE OF THE
C      SHAPE FACTOR.
C
      DIMENSION IA(10),C(10),IBPT(4),FACT(10)
      COMMON ANOP,WTH,HEHT,DEPTH,WTHP,HEHTP,DEPTHP,DUAL,ALAMDA,
     *VVEL,HVEL,RATIO,OPERN1,OPERN2,BCOST,RLCOST,SCOST1,SCOST2,RCOST,
     *C,ATMAR,RITR,FACT,PGIVA,RLBCOST,SRMCOST,BMCOST,
     *IPULL,IADD,NOPR,IA,IBPT,ISTM,IBLDG,ITPUT,INFES,IRACK
      HTT=(WTHP*ICOL1)/HVEL
      VTTT=(HEHTP*NOL1)/VVEL
      BB=VTTT/HTT
      BASE=HTT
      IF(BB.LE.1.00)GO TO 2
      BB=HTT/VTTT
      BASE=VTTT
2     RST=(BB**2/3.0)+1.0
      ST=(RST*BASE)+OPERN1+OPERN2
      RXSQ=((BB*BB*BB*2.0)/3.0)+1.3333
      RVST=RXSQ-(RST**2)
      VST=RVST*BASE**2
      EXSSQ=VST+ST**2
      RDT=(0.5*BB*BB)-(BB*BB*BB/30.0)+1.3333
      DT=(RDT*BASE)+(2*OPERN1)+(2*OPERN2)
      RVDT=((0.3588-0.1321*BB)*RDT)**2
      VDT=RVDT*BASE**2
      EXDSQ=VDT+DT**2
      ETT=((DUAL*(DT/2.0))+((1.0-DUAL)*ST))/60.0
      VTT=((DUAL*(EXDSQ/2.0))+((1.0-DUAL)*EXSSQ)-
     *ETT**2)/3600.0
      CHK=FLOW*ETT
      WTIME=(FLOW*(VTT+(ETT**2)))/(2.0*(1.0-CHK))
      IF(CHK.GE.1.0)WTIME=99999.0
      COMP=1.0/ETT
      OTHER=1.0/(ETT+WTIME)
      IF(ITPUT.EQ.1)GO TO 3
      COMP=OTHER
      OTHER=1.0/ETT
5     FORMAT(3X,'YAVUZ',2I7,2X,9F10.2,'BOZER')
3     RETURN
      END
      SUBROUTINE PWORTH(NOAI,QLAND,QBLDG,QRACK,QSR,RECUR,YY,PWC)
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C
C     GIVEN THE VALUES OF THE COST ELEMENTS, THIS SUBROUTINE
C     WILL COMPUTE THE PRESENT WORTH COST OF THE SYSTEM FOR
C     THE PARTICULAR DESIGN UNDER QUESTION. THE APPROACH
C     DEPENDS ON WHETHER THE BUILDING IS RACK SUPPORTED OR NOT.
C
      DIMENSION IA(10),C(10),IBPT(4),FACT(10)
      COMMON ANOP,WTH,HEHT,DEPTH,WTHP,HEHTP,DEPTHP,DUAL,ALAMDA,
     *VVEL,HVEL,RATIO,OPERN1,OPERN2,BCOST,RLCOST,SCOST1,SCOST2,RCOST,
     *C,ATMAR,RITR,FACT,PGIVA,RLBCOST,SRMCOST,BMCOST,
     *IPULL,IADD,NOPR,IA,IBPT,ISTM,IBLDG,ITPUT,INFES,IRACK
      T1=QRACK+QSR
      IF(IBLDG.EQ.1)GO TO 3
      T1=T1+QBLDG
      SUM=0.0
      DO 2 K2=1,10
2     SUM=SUM+(((10-(K2-1))/55.0)*T1*FACT(K2))
      RITC=(T1-(0.5*SCOST2*NOAI))*0.10
      YY=(2.0*RITC)-25000.0
      PWC=T1+(QLAND*(1-FACT(10)))-(SUM*RITR)-RITC+(RECUR*PGIVA*
     *(1.0-RITR))
      GO TO 4
3     SUM=0.0
      TDEP=0.0
      DO 5 K5=1,10
      R1=((10-(K5-1))/55.0)*T1
      DEP=((40-(K5-1))/820.0)*QBLDG
      TDEP=TDEP+DEP
5     SUM=SUM+((DEP+R1)*FACT(K5))
      RITC=(T1-(0.5*SCOST2*NOAI))*0.10
      YY=(2.0*RITC)-25000.0
      PWC=T1+QBLDG+(QLAND*(1-FACT(10)))-(SUM*RITR)-RITC
     *-((QBLDG-TDEP)*FACT(10))+(RECUR*PGIVA*(1.0-RITR))
4     RETURN
      END
      SUBROUTINE DTRAV(NOL2,ICOL2,COMP,OTHER,SC,DC,BB,
C
C     GIVEN THE RACK SHAPE AND TRAVEL VELOCITIES, THIS
C     SUBROUTINE RETURNS THE MEAN AND VARIANCE OF SINGLE
C     AND DUAL COMMAND TRAVEL TIMES UNDER 'DEDICATED'
C     STORAGE. IT IS ASSUMED THAT PRODUCT CLASSES ARE
C     ENTERED IN 'DESCENDING' C/A RATIOS. IF DESIRED, THIS
C     SUBROUTINE WILL SIMULATE THE RACK INSTEAD OF FOLLOWING
C     THE COMPLETE ENUMERATION SCHEME. IT ALSO RETURNS THE
C     OVER-ALL MEAN AND VARIANCE OF S/R TRAVEL TIME AND
C     THE VALUE OF THE SHAPE FACTOR.
C
     *CAPC,DELT,AC,MIA,TNC)
      DIMENSION TIME(50,300),IPRD(50,300),IA(10),C(10),MIA(10),
     *AC(10),IBPT(4),FACT(10),TNC(10)
      COMMON ANOP,WTH,HEHT,DEPTH,WTHP,HEHTP,DEPTHP,DUAL,ALAMDA,
     *VVEL,HVEL,RATIO,OPERN1,OPERN2,BCOST,RLCOST,SCOST1,SCOST2,RCOST,
     *C,ATMAR,RITR,FACT,PGIVA,RLBCOST,SRMCOST,BMCOST,
     *IPULL,IADD,NOPR,IA,IBPT,ISTM,IBLDG,ITPUT,INFES,IRACK
      TOTM=0.0
      TOTS=0.0
      PHET=-HEHTP/2.0
      DO 12 JM=1,NOL2
      PHET=PHET+HEHTP
      FWT=WTHP/2.0
      DO 12 JC=1,ICOL2
      TH=FWT/HVEL
      TV=PHET/VVEL
      TIME(JM,JC)=TH
      IF(TV.GT.TH)TIME(JM,JC)=TV
      FWT=FWT+WTHP
12    IPRD(JM,JC)=0
C
C
      DO 16 NP=1,NOPR
      CUTM=0.0
      CUTS=0.0
      ILG=MIA(NP)
      DO 19 MN=1,ILG
      TMINI=10000.0
```

```
      LX=1
      LY=1
      DO 17 ILE=1,NOL2
      DO 17 ICO=1,ICOL2
      IF(IPRD(ILE,ICO).NE.0) GO TO 17
      IF(TIME(ILE,ICO).GE.TMINI) GO TO 17
      TMINI=TIME(ILE,ICO)
      LX=ILE
      LY=ICO
17    CONTINUE
      IPRD(LX,LY)=NP
      CUTM=CUTM+TIME(LX,LY)
      CUTS=CUTS+(TIME(LX,LY)*TIME(LX,LY))
19    CONTINUE
      TOTM=TOTM+((AC(NP)/ILG)*CUTM)
      TOTS=TOTS+((AC(NP)/ILG)*CUTS)
16    CONTINUE
      SC=(TOTM*(2.0/CAPC))+OPERN1+OPERN2
      VTS=((4.0/CAPC)*TOTS)-((4/(CAPC*CAPC))*TOTM*TOTM)
      EXSSQ=VTS+SC**2
      STAM=0.0
      RTAM=0.0
      NSL=ICOL2*NOL2
      IF(IFULL.EQ.0)GO TO 2
      NS1=NSL-1
      DO 88 L6=1,NS1
      JPR=((L6-0.00001)/ICOL2)+1.0
      JP=L6-((JPR-1)*ICOL2)
      L7=L6+1
      DO 88 L8=L7,NSL
      KG=((L8-0.00001)/ICOL2)+1.0
      KLM=L8-((KG-1)*ICOL2)
      SLT=(ABS((KLM-JP)*WTHP))/HVEL
      VTTM=(ABS((KG-JPR)*HEHTP))/VVEL
      IF(VTTM.GT.SLT) SLT=VTTM
      TTAYM=TIME(JPR,JP)+TIME(KG,KLM)+SLT
      MA1=IPRD(JPR,JP)
      MA2=IPRD(KG,KLM)
      IF(MA1*MA2)49,88,49
49    BOL=MIA(MA1)*MIA(MA2)*DELT
      RTAM=RTAM+((TTAYM*TNC(MA1)*TNC(MA2))/BOL)
      STAM=STAM+((TTAYM*TTAYM*TNC(MA1)*TNC(MA2))/BOL)
88    CONTINUE
      GO TO 99
2     WEIT=0.0
C
C      'WEIT' IS USED TO KEEP TRACK OF THE PROBABILITIES
C      ASSOCIATED WITH INDIVIDUAL DUAL TRIPS WHICH ARE
C      FORMED BY CHOOSING TWO OPENINGS RANDOMLY
C
      NN=10000
      IF(NSL.GE.10000)NN=15000
C
C      START SIMULATION IN ORDER TO FIND DUAL CYCLE MEAN
C      AND VARIANCE. NOTE THAT SIMULATION TIME= 'NN'
C
      DO 20 K20=1,NN
      JPR=(RANF(X)*NOL2)+0.4999
```

```
      JP=(RANF(X)*ICOL2)+0.4999
      KG=(RANF(X)*NOL2)+0.4999
      KLM=(RANF(X)*ICOL2)+0.4999
      SLT=(ABS((KLM-JP)*WTHP))/HVEL
      VTTM=(ABS((KG-JPR)*HEHTP))/VVEL
      IF(VTTM.GT.SLT) SLT=VTTM
      TTAYM=TIME(JPR,JP)+TIME(KG,KLM)+SLT
      MA1=IPRD(JPR,JP)
      MA2=IPRD(KG,KLM)
      IF(MA1*MA2)79,20,79
79    BOL=MIA(MA1)*MIA(MA2)*DELT
      PROB=(TNC(MA1)*TNC(MA2))/BOL
      RTAM=RTAM+(TTAYM*PROB)
      STAM=STAM+(TTAYM*TTAYM*PROB)
      WEIT=WEIT+PROB
20    CONTINUE
C
C      COMPUTE THE WEIGHTED AVERAGE OF THE TRIP TIMES
C      GENERATED THROUGH SIMULATION
C
      RTAM=RTAM/WEIT
      STAM=STAM/WEIT
99    DC=RTAM+(2*OPERN1)+(2*OPERN2)
      VDT=STAM-(RTAM*RTAM)
      EXDSQ=VDT+DC**2
      BB=TIME(1,ICOL2)/TIME(NOL2,1)
      IF(BB.GE.1.0)BB=1.0/BB
      ETT=((DUAL*(DC/2.0))+((1.0-DUAL)*SC))/60.0
      VTT=((DUAL*(EXDSQ/2.0))+((1.0-DUAL)*EXSSQ)-
     *ETT**2)/3600.0
      CHK=CAPC*ETT
      WTIME=(CAPC*(VTT+(ETT**2)))/(2.0*(1.0-CHK))
      IF(CHK.GE.1.0)WTIME=99999.0
      COMP=1.0/ETT
      OTHER=1.0/(ETT+WTIME)
      IF(ITPUT.EQ.1)GO TO 5
      COMP=OTHER
      OTHER=1.0/ETT
5     RETURN
      END
```

APPENDIX 12

Outputs related to the sensitivity analysis on "DUAL"

```
    GEORGIA TECH. -- DESIGN OF AN AS/RS

    TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 48. 48. 48.
    TYPE UNIT LOAD WEIGHT(LB.S)
? 2000.
    TYPE DUAL COMMAND PERCENTAGE
? 0.0
    TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 60. 240.
    TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.25 0.25
    TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 25. 6.
    TYPE 0,1,2 OR 3 FOR S/R LOGIC
      0=MAN-ON-BOARD  1=ON THE S/R
      2=OFF THE S/R  3=CENTRAL CONSOLE
? 2
    TYPE ANNUAL MAINTENANCE COST/SR
? 2200.
    TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 5.5
    TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 0
    TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 1
    TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.10 0.50
    TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 0
    TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)
? 6000.
    TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)
? 145.
```

```
NO. OF AISLES      5
NO. OF LEVELS       11.00
NO. OF COLUMNS       55.00

BLDG. LENGTH      290.67
BLDG. WIDTH       67.50
BLDG. HEIGHT       57.17

LAND COST 117720.00
BLDG. COST 648655.54
RACK COST 502036.82
S/R MACHINE COST 520000.00    DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST    107910.00
S/R MAINTENANCE COST     11000.00

SYSTEM THROUGHPUT(0)      19.76 OP.NS/HR.
SYSTEM THROUGHPUT(1)     165.36 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     1.81    2.77
THE SHAPE FACTOR IS          .83

TOTAL NO. OF OPENINGS   6050.00


PW COST OF ABOVE DESIGN IS    1365464.72 DOLLARS
(TAX LIABILITY SHOULD BE > 289638.47)
```

```
GEORGIA TECH. - DESIGN OF AN AS/RS

  TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 48. 48. 48.
  TYPE UNIT LOAD WEIGHT(LB.S)
? 2000.
  TYPE DUAL COMMAND PERCENTAGE
? 0.2
  TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 60. 240.
  TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.25 0.25
  TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 25. 6.
  TYPE 0,1,2 OR 3 FOR S/R LOGIC
    0=MAN-ON-BOARD  1=ON THE S/R
    2=OFF THE S/R  3=CENTRAL CONSOLE
? 2
  TYPE ANNUAL MAINTENANCE COST/SR
? 2200.
  TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 5.5
  TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 0
  TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 1
  TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.10 0.50
  TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 0
  TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)
? 6000.
  TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)
? 145.
```

NO. OF AISLES 5
NO. OF LEVELS 11.00
NO. OF COLUMNS 55.00

BLDG. LENGTH 290.67
BLDG. WIDTH 67.50
BLDG. HEIGHT 57.17

LAND COST 117720.00
BLDG. COST 648655.54
RACK COST 502036.82
S/R MACHINE COST 520000.00 DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST 107910.00
S/R MAINTENANCE COST 11000.00

SYSTEM THROUGHPUT(0) 26.28 OP.NS/HR.
SYSTEM THROUGHPUT(1) 173.55 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES 1.81 2.77
THE SHAPE FACTOR IS .83

TOTAL NO. OF OPENINGS 6050.00

PW COST OF ABOVE DESIGN IS 1365464.72 DOLLARS
(TAX LIABILITY SHOULD BE > 289638.47)

```
GEORGIA TECH. - DESIGN OF AN AS/RS

  TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 48. 48. 48.
  TYPE UNIT LOAD WEIGHT(LB.S)
? 2000.
  TYPE DUAL COMMAND PERCENTAGE
? 0.4
  TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 60. 240.
  TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.25 0.25
  TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 25. 6.
  TYPE 0,1,2 OR 3 FOR S/R LOGIC
    0=MAN-ON-BOARD  1=ON THE S/R
    2=OFF THE S/R  3=CENTRAL CONSOLE
? 2
  TYPE ANNUAL MAINTENANCE COST/SR
? 2200.
  TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 5.5
  TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 0
  TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 1
  TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.1 0.5
  TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 0
  TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)
? 6000.
  TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)
? 145.
```

```
NO. OF AISLES      5
NO. OF LEVELS       11.00
NO. OF COLUMNS       55.00

BLDG. LENGTH     290.67
BLDG. WIDTH      67.50
BLDG. HEIGHT      57.17

LAND COST 117720.00
BLDG. COST 648655.54
RACK COST 502036.82
S/R MACHINE COST 520000.00   DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST   107910.00
S/R MAINTENANCE COST    11000.00

SYSTEM THROUGHPUT(0)      32.81 OP.NS/HR.
SYSTEM THROUGHPUT(1)     182.59 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     1.81   2.77
THE SHAPE FACTOR IS        .83

TOTAL NO. OF OPENINGS   6050.00


PW COST OF ABOVE DESIGN IS   1365464.72 DOLLARS
(TAX LIABILITY SHOULD BE > 289638.47)
```

```
GEORGIA TECH. - DESIGN OF AN AS/RS


 TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 48. 48. 48.
 TYPE UNIT LOAD WEIGHT(LB.S)
? 2000.
 TYPE DUAL COMMAND PERCENTAGE
? 0.6
 TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 60. 240.
 TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.25 0.25
 TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 25. 6.
 TYPE 0,1,2 OR 3 FOR S/R LOGIC
   0=MAN-ON-BOARD  1=ON THE S/R
   2=OFF THE S/R  3=CENTRAL CONSOLE
? 2
 TYPE ANNUAL MAINTENANCE COST/SR
? 2200.
 TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 5.5
 TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 0
 TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 1
 TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.10 0.50
 TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 0
 TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)
? 6000.
 TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)
? 145.
```

```
NO. OF AISLES     5
NO. OF LEVELS      11.00
NO. OF COLUMNS      55.00

BLDG. LENGTH     290.67
BLDG. WIDTH      67.50
BLDG. HEIGHT      57.17

LAND COST 117720.00
BLDG. COST 648655.54
RACK COST 502036.82
S/R MACHINE COST 520000.00    DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST    107910.00
S/R MAINTENANCE COST     11000.00

SYSTEM THROUGHPUT(0)       39.38 OP.NS/HR.
SYSTEM THROUGHPUT(1)     192.62 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     1.81    2.77
THE SHAPE FACTOR IS         .83

TOTAL NO. OF OPENINGS    6050.00


PW COST OF ABOVE DESIGN IS    1365464.72 DOLLARS
(TAX LIABILITY SHOULD BE > 289638.47)
```

```
   GEORGIA TECH. - DESIGN OF AN AS/RS

   TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 48. 48. 48.
   TYPE UNIT LOAD WEIGHT(LB.S)
? 2000.
   TYPE DUAL COMMAND PERCENTAGE
? 0.80
   TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 60. 240.
   TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.25 0.25
   TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 25. 6.
   TYPE 0,1,2 OR 3 FOR S/R LOGIC
     0=MAN-ON-BOARD  1=ON THE S/R
     2=OFF THE S/R  3=CENTRAL CONSOLE
? 2
   TYPE ANNUAL MAINTENANCE COST/SR
? 2200.
   TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 5.5
   TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 0
   TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 1
   TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.10 0.50
   TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 0
   TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)
? 6000.
   TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)
? 145.
```

```
NO. OF AISLES      4
NO. OF LEVELS       11.00
NO. OF COLUMNS       69.00

BLDG. LENGTH     356.00
BLDG. WIDTH      54.00
BLDG. HEIGHT      57.17

LAND COST 115344.00
BLDG. COST 635563.41
RACK COST 503862.41
S/R MACHINE COST 416000.00   DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST   105732.00
S/R MAINTENANCE COST     8800.00

SYSTEM THROUGHPUT(0)       1.30 OP.NS/HR.
SYSTEM THROUGHPUT(1)     146.81 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     2.04   3.07
THE SHAPE FACTOR IS          .66

TOTAL NO. OF OPENINGS   6072.00


PW COST OF ABOVE DESIGN IS   1285264.65 DOLLARS
(TAX LIABILITY SHOULD BE > 270485.16)
```

```
GEORGIA TECH. - DESIGN OF AN AS/RS

  TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 48. 48. 48.
  TYPE UNIT LOAD WEIGHT(LB.S)
? 2000.
  TYPE DUAL COMMAND PERCENTAGE
? 1.0
  TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 60. 240.
  TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.25 0.25
  TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 25. 6.
  TYPE 0,1,2 OR 3 FOR S/R LOGIC
    0=MAN-ON-BOARD  1=ON THE S/R
    2=OFF THE S/R  3=CENTRAL CONSOLE
? 2
  TYPE ANNUAL MAINTENANCE COST/SR
? 2200.
  TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 5.5
  TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 0
  TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 1
  TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.10 0.50
  TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 0
  TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)
? 6000.
  TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)
? 145.
```

```
NO. OF AISLES     4
NO. OF LEVELS      11.00
NO. OF COLUMNS      69.00

BLDG. LENGTH     356.00
BLDG. WIDTH      54.00
BLDG. HEIGHT      57.17

LAND COST 115344.00
BLDG. COST 635563.41
RACK COST 503862.41
S/R MACHINE COST 416000.00   DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST   105732.00
S/R MAINTENANCE COST     8800.00

SYSTEM THROUGHPUT(0)       7.64 OP.NS/HR.
SYSTEM THROUGHPUT(1)     156.42 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES    2.04   3.07
THE SHAPE FACTOR IS         .66

TOTAL NO. OF OPENINGS   6072.00


PW COST OF ABOVE DESIGN IS   1285264.65 DOLLARS
(TAX LIABILITY SHOULD BE > 270485.16)
```

APPENDIX 13

Outputs related to the sensitivity analysis on λ(TPUT)

```
GEORGIA TECH. - DESIGN OF AN AS/RS

  TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 48. 48. 48.
  TYPE UNIT LOAD WEIGHT(LB.S)
? 2000.
  TYPE DUAL COMMAND PERCENTAGE
? 0.5
  TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 60. 240.
  TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.25 0.25
  TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 25. 6.
  TYPE 0,1,2 OR 3 FOR S/R LOGIC
    0=MAN-ON-BOARD  1=ON THE S/R
    2=OFF THE S/R  3=CENTRAL CONSOLE
? 2
  TYPE ANNUAL MAINTENANCE COST/SR
? 2200.
  TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 5.5
  TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 0
  TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 1
  TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.10 0.50
  TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 0
  TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)
? 6000.
  TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)
? 100.
```

```
NO. OF AISLES     4
NO. OF LEVELS      11.00
NO. OF COLUMNS      69.00

BLDG. LENGTH     356.00
BLDG. WIDTH     54.00
BLDG. HEIGHT      57.17

LAND COST 115344.00
BLDG. COST 635563.41
RACK COST 503862.41
S/R MACHINE COST 416000.00   DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST    105732.00
S/R MAINTENANCE COST      8800.00

SYSTEM THROUGHPUT(0)      29.26 OP.NS/HR.
SYSTEM THROUGHPUT(1)     134.41 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     2.04   3.07
THE SHAPE FACTOR IS         .66

TOTAL NO. OF OPENINGS   6072.00


PW COST OF ABOVE DESIGN IS   1285264.65 DOLLARS
(TAX LIABILITY SHOULD BE > 270485.16)
```

```
GEORGIA TECH. - DESIGN OF AN AS/RS

  TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 48. 48. 48.
  TYPE UNIT LOAD WEIGHT(LB.S)
? 2000.
  TYPE DUAL COMMAND PERCENTAGE
? 0.50
  TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 60. 240.
  TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.25 0.25
  TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 25. 6.
  TYPE 0,1,2 OR 3 FOR S/R LOGIC
    0=MAN-ON-BOARD  1=ON THE S/R
    2=OFF THE S/R  3=CENTRAL CONSOLE
? 2
  TYPE ANNUAL MAINTENANCE COST/SR
? 2200.
  TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 5.5
  TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 0
  TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 1
  TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.10 0.50
  TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 0
  TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)
? 6000.
  TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)
? 125.
```

```
NO. OF AISLES     4
NO. OF LEVELS      11.00
NO. OF COLUMNS      69.00

BLDG. LENGTH     356.00
BLDG. WIDTH      54.00
BLDG. HEIGHT      57.17

LAND COST 115344.00
BLDG. COST 635563.41
RACK COST 503862.41
S/R MACHINE COST 416000.00   DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST    105732.00
S/R MAINTENANCE COST      8800.00

SYSTEM THROUGHPUT(0)        7.72 OP.NS/HR.
SYSTEM THROUGHPUT(1)     134.41 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     2.04   3.07
THE SHAPE FACTOR IS         .66

TOTAL NO. OF OPENINGS    6072.00


PW COST OF ABOVE DESIGN IS    1285264.65 DOLLARS
(TAX LIABILITY SHOULD BE > 270485.16)
```

```
   GEORGIA TECH. - DESIGN OF AN AS/RS

   TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 48. 48. 48.
   TYPE UNIT LOAD WEIGHT(LB.S)
? 2000.
   TYPE DUAL COMMAND PERCENTAGE
? 0.5
   TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 60. 240.
   TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.25 0.25
   TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 25. 6.
   TYPE 0,1,2 OR 3 FOR S/R LOGIC
     0=MAN-ON-BOARD  1=ON THE S/R
     2=OFF THE S/R  3=CENTRAL CONSOLE
? 2
   TYPE ANNUAL MAINTENANCE COST/SR
? 2200.
   TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 5.5
   TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 0
   TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 1
   TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.10 0.50
   TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 0
   TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)
? 6000.
   TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)
? 150.
```

```
NO. OF AISLES     5
NO. OF LEVELS      11.00
NO. OF COLUMNS      55.00

BLDG. LENGTH     290.67
BLDG. WIDTH      67.50
BLDG. HEIGHT      57.17

LAND COST 117720.00
BLDG. COST 648655.54
RACK COST 502036.82
S/R MACHINE COST 520000.00   DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST   107910.00
S/R MAINTENANCE COST    11000.00

SYSTEM THROUGHPUT(0)      31.68 OP.NS/HR.
SYSTEM THROUGHPUT(1)     107.47 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES    1.81   2.77
THE SHAPE FACTOR IS        .83

TOTAL NO. OF OPENINGS   6050.00


PW COST OF ABOVE DESIGN IS   1365464.72 DOLLARS
(TAX LIABILITY SHOULD BE > 289638.47)
```

```
GEORGIA TECH. - DESIGN OF AN AS/RS

   TYPE THE DEPTH,WIDTH,HEIGHT OF UNIT LOAD(INCHES)
? 48. 48. 48.
   TYPE UNIT LOAD WEIGHT(LB.S)
? 2000.
   TYPE DUAL COMMAND PERCENTAGE
? 0.50
   TYPE VERTICAL,HORIZONTAL VELOCITY OF S/R(FPM)
? 60. 240.
   TYPE PICK-UP,PUT-DOWN TIMES (MIN.S)
? 0.25 0.25
   TYPE UNIT BUILDING,UNIT LAND COST($/FT**2)
? 25. 6.
   TYPE 0,1,2 OR 3 FOR S/R LOGIC
     0=MAN-ON-BOARD  1=ON THE S/R
     2=OFF THE S/R  3=CENTRAL CONSOLE
? 2
   TYPE ANNUAL MAINTENANCE COST/SR
? 2200.
   TYPE ANNUAL BLDG. MAINT. COST/SQ.FT.
? 5.5
   TYPE 0 (RACK) OR 1 (NON-RACK) SUPPORTED BLDG.
? 0
   TYPE 0 TO INCLUDE WAITING TIME,OTHERWISE TYPE 1
? 1
   TYPE ATMARR,APPLICABLE INCOME TAX RATE
? 0.10 0.50
   TYPE 0 FOR RANDOMIZED;1 FOR DEDICATED STORAGE
? 0
   TYPE TOTAL NO. OF SLOTS REQUIRED(REAL)
? 6000.
   TYPE THE REQUIRED THROUGHPUT(OP.NS/HR.)
? 200.
```

```
NO. OF AISLES      6
NO. OF LEVELS       11.00
NO. OF COLUMNS       46.00

BLDG. LENGTH      248.67
BLDG. WIDTH       81.00
BLDG. HEIGHT       57.17

LAND COST 120852.00
BLDG. COST 665913.35
RACK COST 503862.41
S/R MACHINE COST 624000.00   DOLLARS

THE RECURRING COSTS ARE

BLDG. MAINTENANCE COST    110781.00
S/R MAINTENANCE COST     13200.00

SYSTEM THROUGHPUT(0)       34.50 OP.NS/HR.
SYSTEM THROUGHPUT(1)     240.94 OP.NS/HR.

EXP. SINGLE & DUAL COMMAND TRAVEL TIMES     1.69   2.60
THE SHAPE FACTOR IS         .99

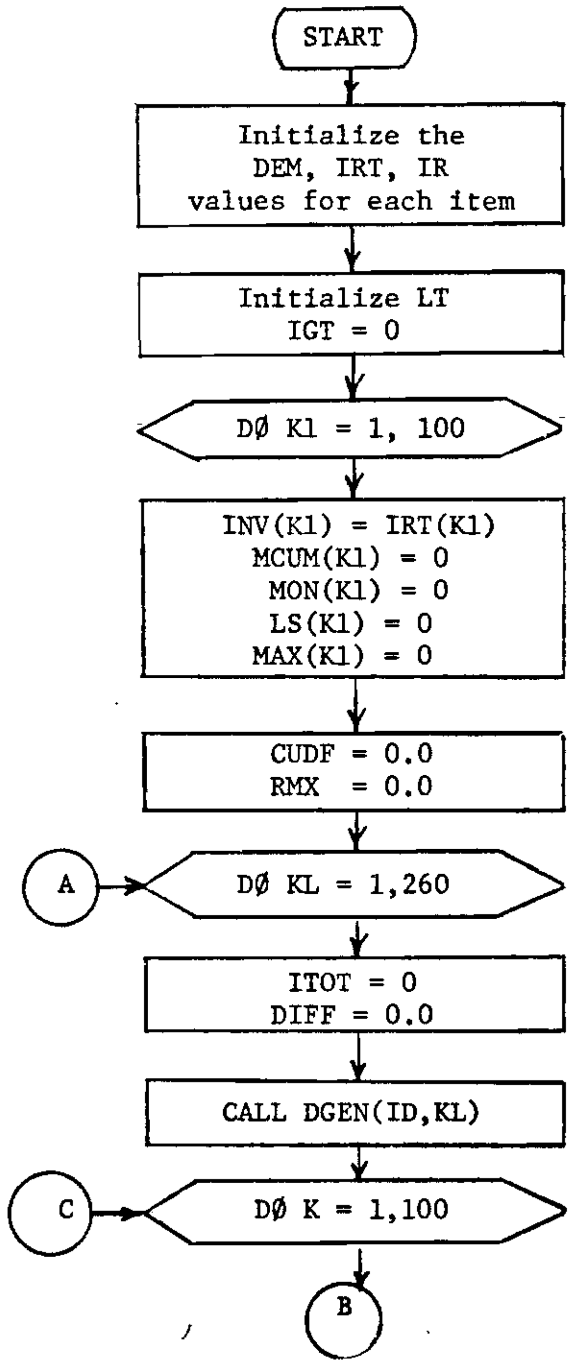
TOTAL NO. OF OPENINGS   6072.00

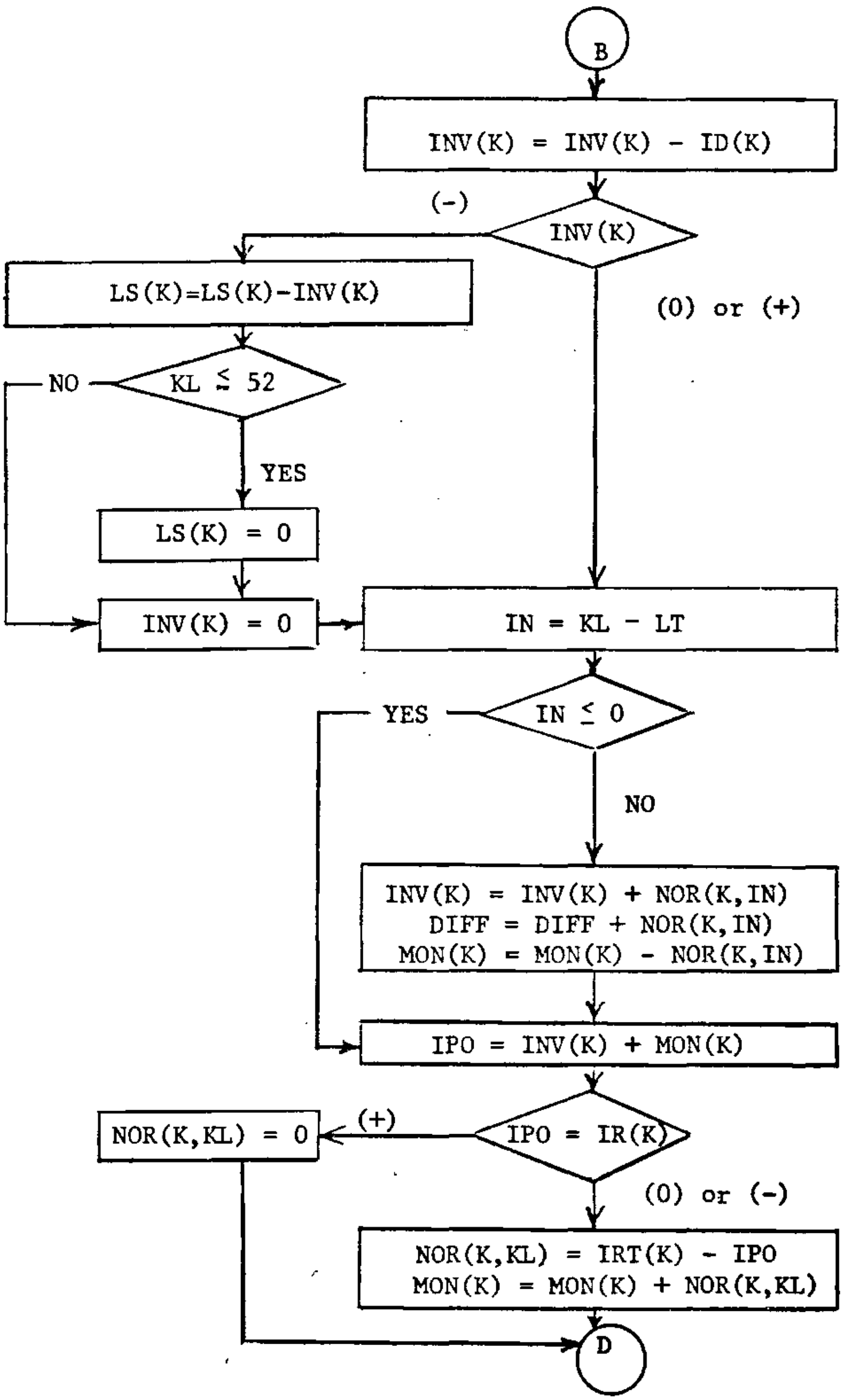

PW COST OF ABOVE DESIGN IS   1452553.83 DOLLARS
(TAX LIABILITY SHOULD BE > 310355.15)
```

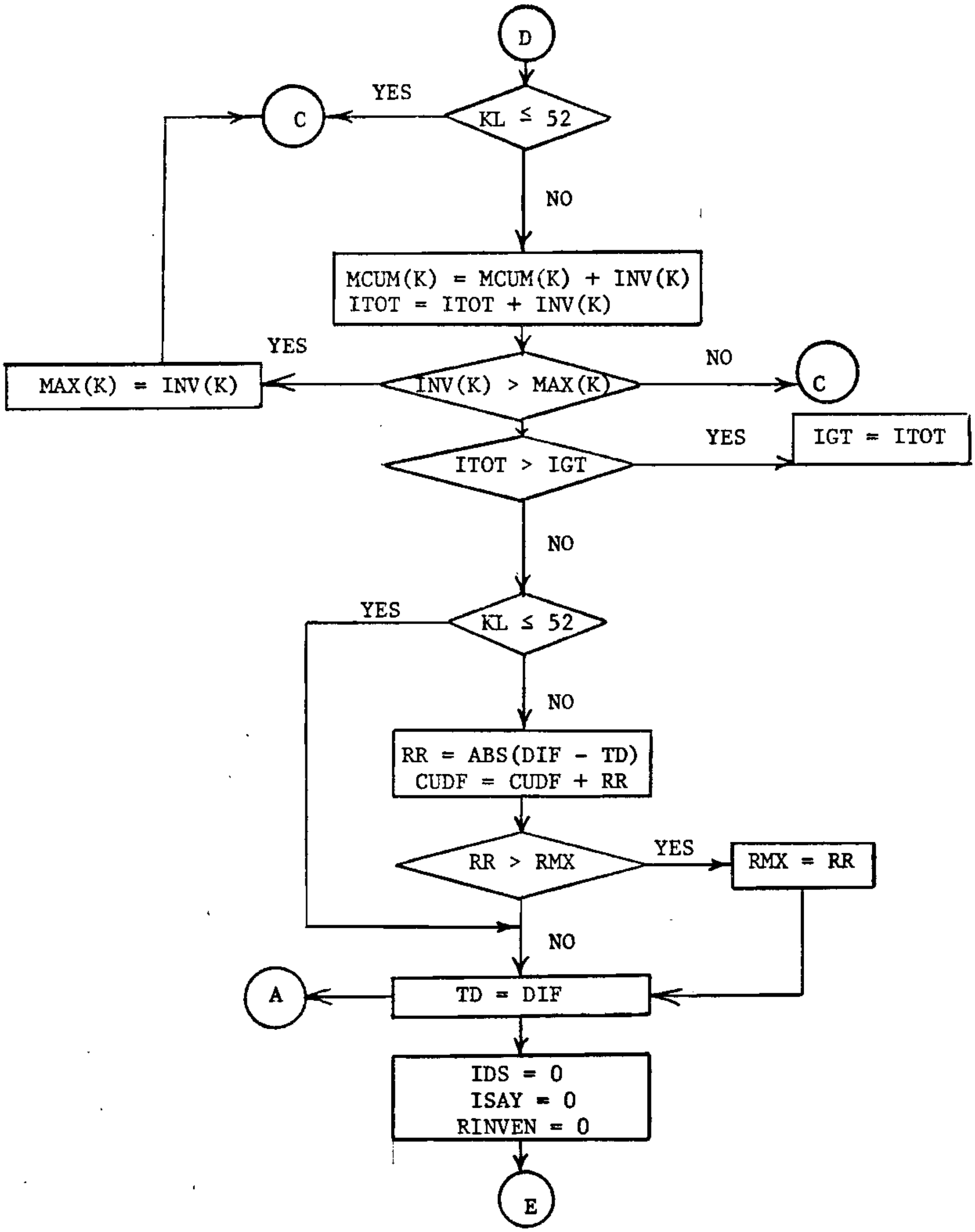
APPENDIX 14

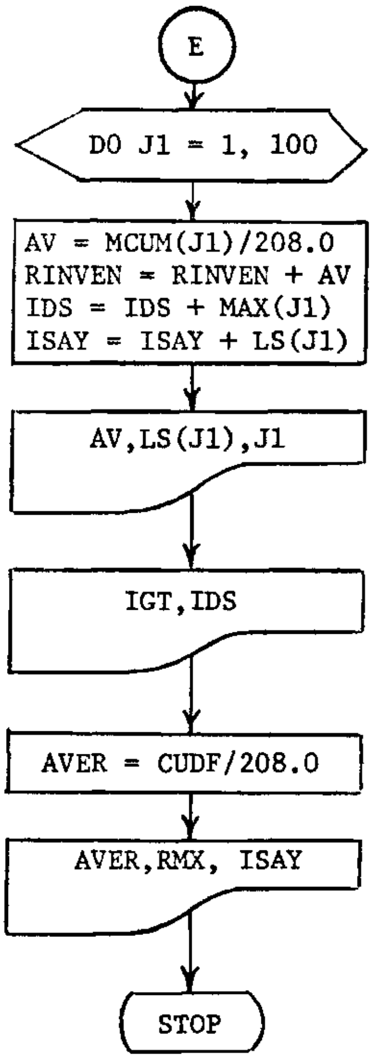
Detailed flow-chart of the program listing

presented in Appendix 1


START
Initialize the
DEM, IRT, IR
values for each item
Initialize LT
IGT = 0
DØ K1 = 1, 100
INV(K1) = IRT(K1)
MCUM(K1) = 0
MON(K1) = 0
LS(K1) = 0
MAX(K1) = 0
CUDF = 0.0
RMX = 0.0
A
DØ KL = 1,260
ITOT = 0
DIFF = 0.0
CALL DGEN(ID,KL)
C
DØ K = 1,100
B


B
INV(K) = INV(K) - ID(K)
(-)
INV(K)
(0) or (+)
LS(K)=LS(K)-INV(K)
NO
KL ≤ 52
YES
LS(K) = 0
INV(K) = 0
IN = KL - LT
YES
IN ≤ 0
NO
INV(K) = INV(K) + NOR(K,IN)
DIFF = DIFF + NOR(K,IN)
MON(K) = MON(K) - NOR(K,IN)
IPO = INV(K) + MON(K)
NOR(K,KL) = 0
(+)
IPO = IR(K)
(0) or (-)
NOR(K,KL) = IRT(K) - IPO
MON(K) = MON(K) + NOR(K,KL)
D


D
KL ≤ 52
YES
C
NO
MCUM(K) = MCUM(K) + INV(K)
ITOT = ITOT + INV(K)
INV(K) > MAX(K)
YES
MAX(K) = INV(K)
NO
C
ITOT > IGT
YES
IGT = ITOT
NO
KL ≤ 52
YES
NO
RR = ABS(DIF - TD)
CUDF = CUDF + RR
RR > RMX
YES
RMX = RR
NO
TD = DIF
A
IDS = 0
ISAY = 0
RINVEN = 0
E


E
DO J1 = 1, 100
AV = MCUM(J1)/208.0
RINVEN = RINVEN + AV
IDS = IDS + MAX(J1)
ISAY = ISAY + LS(J1)
AV,LS(J1),J1
IGT,IDS
AVER = CUDF/208.0
AVER,RMX, ISAY
STOP

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* The following were not cited, but were useful in the conduct of the research.